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BOOK REVIEWS

Toponogov, V.A., *Differential Geometry of Curves and Surfaces, A Concise Guide*, Birkhäuser, 2006, paperback, 206 pp., ISBN 0-8176-4384-2.

The differential geometry of curves and surfaces is one of the most venerable parts of mathematics which is still present in the curriculum. It would seem, therefore, there is not much left to be said. From time to time, however, some surprises come to light. This book, written by one of the finest Russian geometers, a well-known expert in Riemannian geometry, is such an example.

Despite the modest size and title, the book is more than just a "concise guide" to the differential geometry of curves and surfaces.

The parts of the book are the classical ones: curves, extrinsic geometry of surfaces and intrinsic geometry of surfaces. The novelty of the book doesn't reside in the language (which is the classical one), but rather in the organization of material. The basic notions and results are treated very rapidly (whence, probably, the title), but then a wealth of material is exposed under the, somehow misleading, name of "Problems". The sections carrying this name are, in fact, collections of very interesting results, usually with full proofs. Some of the results are well-known, others are new. All of them, however, are looked from a fresh perspective and, sometime, new proofs are provided for classical results.

I found particularly illuminating the last chapter, on the intrinsic geometry of surfaces, in fact a compact introduction to 2-dimensional Riemannian geometry. A special mention, of course, deserve the final sections, on comparison theorems, which was one of the fields of expertise of Victor Toponogov.

All in all, this is a very remarkable book, and I'm sure it will be highly appreciated both by students and teachers. It is a pity that the author didn't live to see it printed (he passed away in 2004).

Paul Blaga

Cohen, R.L., Hess, K. and Voronov, A., *String Topology and Cyclic Homology*, Birkhäuser (Advanced Courses in Mathematics, CRM, Barcelona), 2006, paperback, 163 pp., ISBN-10: 3-7643-2182-2, ISBN-13: 978-2-7643-2182-6.

This book is based on the lectures delivered by the authors on string topology and cyclic homology, held in Almeria, Spain, from September 16 to 20, 2003.

The first part of the book, written by the first and the last author, is devoted to string topology, a recent theory, initiated by Chas and Sullivan in 1999, in order to investigate the differential and algebraic topological properties of the spaces of loops and paths on a manifold.

The authors provide both a comprehensive introduction to the subject and a description of the state of the art in the field. They start by an exposition of the intersection theory in loop spaces, which is the very core of the string topology, and continue by taking different approaches to the subject (operads, field theoretical approach, Morse theoretical viewpoint). The final chapter is devoted to brane topology.

In the second part of the book, written by K. Hess, there are given computational methods for the so-called *topological cyclic homology*, a variant of the classical cyclic homology,introduced by Bökstedt, Hsiang and Madsen. More specifically, it provides an algebraic model for the Mod 2 topological cyclic homology. The subjects treated are connected in more than one way to the the ones examined in the first part of the book.

This is a high level book, having as target people interested in the modern aspects of algebraic topology, from advanced graduate student to researcher. It covers material that cannot be found, in monograph form, elsewhere. The style is, however, comprehensive and the book can be read without major difficulties by someone with the right prerequisites (algebraic topology, differential topology, homological algebra). It can be useful, also, to physicists interested in string theory.

Paul Blaga

Clark, J., Lomp, C., Vanaja, N. and Wisbauer, R., *Lifting Modules. Supplements and projectivity in module theory*, Birkhäuser Verlag, Basel, Boston, Berlin, 2006, XII+394 pp., Softcover, ISBN 10: 3-7643-7572-8, ISBN 13: 978-3-7643-7572-0.

The theory of extending modules and lifting modules have been developed very much in the last two decades, due to its special importance in ring and module theory. Some of these results have been extremely useful tools; for instance, the Smith-Osofsky theorem made possible to obtain solutions to old problems, unaccessible before by other means. Extending modules generalize injective modules and, dually to some extent, lifting modules generalize projective supplemented modules. Meanwhile the main results on extending modules were already contained in a monograph, namely *Extending modules* by N.V. Dung, D.V. Huynh, P.F. Smith and R. Wisbauer, those on lifting modules were still to be gathered and unified. The present monograph comes to fill that gap in the literature, offering an up-to-date account on lifting modules.

The book is structured in 5 chapters and a short appendix on two graph-theoretical techniques. The first chapter, *Basic notions*, contains some general results on injectivity, small modules, coclosed submodules, projectivity and hollow dimension. Chapter 2, *Preradicals and torsion theories*, gives the needed material on preradicals, torsion theories and proper classes, with emphasis on the discussion of small modules and relative supplements. The third chapter, *Decompositions of modules*, deals with the exchange property, local direct summands, total and LE-decompositions, cancellation and decomposition uniqueness. In the last preparatory chapter, *Supplements in modules*, there are established the main properties on semilocal and weakly supplemented modules, semilocal endomorphism rings, supplemented modules and submodules with unique coclosure. The core of the book and the most extensive chapter is Chapter 5, *From lifting modules to perfect modules*. It offers an exhaustive investigation of the properties of lifting modules, dealing with topics such as finite direct sums of lifting modules, the lifting property for infinite direct sums, Σ-lifting modules, semi-discrete and quasi-discrete modules, discrete and perfect modules, injective modules lifting in $\sigma[M]$ and extending modules lifting in $\sigma[M]$.

The book is well written, with numerous examples, and the reader is taken thoroughly into the specialized theory of lifting modules. I consider this volume as a valuable tool and source of inspiration both for the student interested in a future research in an active field of algebra and for the working researcher.

Septimiu Crivei

Kurdachenko, L.A., Otal, J. and Subbotin, I.Ya., *Artinian Modules over Group Rings*, Birkhäuser Verlag, Basel - Boston - Berlin, 2007, XII+247 pp., Softcover, ISBN 10: 3-7643-7764-X, ISBN 13: 978-3-7643-7764-9.

It is an old idea to to study a finite group G by letting it act on finite dimensional vector spaces over a field F. This approach leads to the consideration of finite dimensional modules over the group algebra FG, and it can be naturally extended to study infinite groups which are closely related to finite groups. The authors present here recent developments on artinian modules over group rings of generalized nilpotent groups.

The volume under review is divided into 17 chapters. Let me briefly present the content of each one. *1. Modules with chain conditions.* Some basic concepts and results are introduced, and some important problems of the theory are already formulated. *2. Ranks of groups.* Several numerical invariants of groups are introduced and discussed: the 0-rank, the torsion-free rank, the Hirsch number, the p-rank, the reduced rank, the special (Prüfer–Maltsev) torsion-free rank. Also, several classes of groups are defined by using these invariants. *3. Some generalized nilpotent groups.* Special subgroups of a group are defined with respect to a formation of groups, and generalizations of nilpotent groups are discussed. *4. Artinian modules and the socle.* The socle of a module, and related concepts such as socular height are studied. *5. Reduction to subgroups of finite index.* If A is an artinian (noetherian) module over the group ring RG and H is a normal subgroup of finite index of G, then A is an artinian (noetherian) module over RH. This phenomenon of inheritance is the theme of this chapter. *6. Modules over Dedekind domain.* There are many results in the literature on this topic, and the authors present here those which are used in subsequent chapters. *7. The Kovacs–Newman theorem.* The following problem is discussed. Assume that the RG-submodule B of A has an R-complement. Under which conditions have B an RG-complement. This question originates from Maschke's theorem, and has applications to semisimplicity criteria. *8. Hartley's classes of modules.* Semisimplicity criteria for a specific class of modules that is wider than artinian modules over Chernikov groups are discussed. *9. The injectivity of some simple classes.* Results on this topic by D.R. Farkas and R.L. Snider and by B. Hartley are presented. *10. Direct decompositions of artinian modules.* The so-called Baer decomposition of and RG-modules with respect to a formation of groups is investigated. *11. On the countability of artinian modules over FC-hypercentral groups.* This chapter deals with the following question. What can be said about a countable group G if an artinian DG-module (where D is a Dedekind domain) is countably generated over D. *12. Artinian modules over periodic abelian groups.* Results on finiteness conditions and decomposability of modules over periodic abelian groups of finite special rank are given. *13. Nearly injective modules.* Let A be a simple $\mathbb{Z}G$-module and let E be a $\mathbb{Z}G$-injective envelope of A. Inside E consider the $\mathbb{Z}$-injective envelope U of A.

In which cases is E equal to U? *14. Artinian modules over abelian groups of finite section ranks.* Results of L.A. Kurdachenko on the description of injective envelopes of artinian modules and on the description of artinian modules over a group G such that $|G/G^p| \leq p$ are presented. *15. The injective envelopes of simple modules over group rings.* This chapter is motivated by the following question. When is an injective envelope of an artinian module likewise artinian? *16. Quasifinite modules.* The focus is on DG-modules A satisfying the following conditions (D is a Dedekind domain): A is not finitely generated as a D-module; if B is a proper DG-submodule of A, then B is finitely generated as a D-module; A is the union of its proper DG-submodule. *17. Some applications: splitting over the locally nilpotent residual.* Let A be a $\mathbb{Z}G$-module. The main result here is a theorem of D.J.S. Robinson which gives conditions under which every extension of A by G splits over A, and all complements of A in such an extension are conjugate.

The book ends with a rich bibliography containing 313 titles (note that extensive bibliographical comments are included in each chapter), and a detailed index. The content is highly specialized, reflecting the research interests of the authors, but presented in a self-contained and readable manner. Therefore, this volume is highly recommended to researchers and graduate students alike.

Andrei Marcus