

THE FEKETE-SZEGŐ INEQUALITY FOR A SUBCLASS OF ANALYTIC FUNCTIONS INVOLVING HADAMARD PRODUCT

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Abstract. For $0 < \alpha \leq 1$, $0 \leq \beta \leq \lambda \leq 1$, $0 \leq \delta < 1$, $0 \leq \nu < 1$ and $\rho > 0$, let $\mathfrak{R}(\Phi, \Psi; \lambda, \beta, \alpha, \delta, \nu, \rho)$ be the class of analytic functions defined in the open unit disk E by

$$\left| \arg \left(\frac{\lambda\beta z^3 f'''(z) + (2\lambda\beta + \lambda - \beta)z^2 f''(z) + z f'(z)}{\lambda\beta z^2 g''(z) + (\lambda - \beta)z g'(z) + (1 - \lambda + \beta)f(z)} - \delta \right) \right| < \frac{\pi\alpha}{2}, \quad (z \in E)$$

where $g(z) = z + b_2 z^2 + b_3 z^3 + \dots$ is analytic function on E and satisfies

$$\left| \arg \left(\frac{g(z) * \Phi(z)}{g(z) * \Psi(z)} - \nu \right) \right| < \frac{\pi\rho}{2}, \quad (z \in E)$$

for some $\Phi(z) = z + \sum_{n=2}^{\infty} \Upsilon_n z^n$ and $\Psi(z) = z + \sum_{n=2}^{\infty} \gamma_n z^n$ analytic in E such that $g(z) * \Psi(z) \neq 0$, $\Upsilon_n \geq 0$, $\gamma_n \geq 0$ and $\Upsilon_n > \gamma_n (n \geq 2)$. For $f \in \mathfrak{R}(\Phi, \Psi; \lambda, \beta, \alpha, \delta, \nu, \rho)$ and given by $f(z) = z + a_2 z^2 + a_3 z^3 + \dots$, a sharp upper bound is obtained for functional $|a_3 - \mu a_2^2|$ when $\mu \geq 1$.

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