

INVERSE PROBLEM OF DYNAMICS IN NON-FLAT SPACES:
SOLVABLE CASES OF THE TWO BASIC EQUATIONS

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Abstract. We study several solvable cases of a first-order partial differential equation given by Mertens (1981). This equation combines the potential function $V = V(u, v)$ with a mono-parametric family of regular orbits $f(u, v) = c$ on a given surface S submersed in \mathbb{E}^3 and the function of energy-dependence $E = E(f)$ is given in advance. In the generic case it is shown that two differential conditions must be hold for the “slope function” $\gamma = f_v/f_u$ in order the above equation has solution. Moreover, in the above solvable cases, the second order PDE given by Bozis and Mertens (1985) reduces to its canonical form and can be solved too. Pertinent examples are given.

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