

A LAGRANGIAN RELAXATION APPROACH TO THE GENERALIZED MINIMUM SPANNING TREE PROBLEM

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Abstract. The Generalized Minimum Spanning Tree Problem, denoted GMST, is a variant of the classical Minimum Spanning Tree problem, and consists of finding a minimum-cost tree spanning a subset of nodes which includes exactly one node from every cluster in an undirected graph whose nodes are partitioned into clusters and whose edges are defined between nodes belonging to different clusters. The GMST problem is *NP-hard* even when defined on trees. In this paper we consider an approach based on Lagrangian relaxation of the bidirectional flow formulation of the GMST problem. The subgradient method is used to obtain lower bounds. Computational results are reported for many instances of the problem.

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Key words. Minimum spanning tree, generalized minimum spanning trees, *NP-hard*, Lagrangian relaxation, subgradient method.

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