

EXISTENCE OF INVARIANT APPROXIMATION RESULT
IN LOCALLY CONVEX SPACE

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Abstract. A fixed point theorem of Hadzic [5] is generalized to locally convex spaces and the new result is applied to extend a recent result on invariant approximation of Jungck and Sessa [8] and Mukherjee and Som [11] for non-convex condition of domain and without affineness condition of mappings. Some known results [1], [6], [13] and [16] are also extended and improved. A property known as *Property Γ* is defined to restore the affineness nature of mapping.

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