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MONOTONICITY, COMPARISON AND MINKOWSKI'S INEQUALITY FOR GENERALIZED MUIRHEAD MEANS IN TWO VARIABLES

TIBERIU TRIF

Abstract. Given the real numbers a and b with $a+b\neq 0$, the generalized Muirhead (or symmetric) mean with parameters a, b is the function $\Sigma_{a,b}(\cdot,\cdot)$, defined by

$$\Sigma_{a,b}(x,y)=\left(\frac{x^a y^b+x^b y^a}{2}\right)^{\frac{1}{a+b}}.$$

The aim of the paper is to investigate the monotonicity of $\Sigma_{a,b}$ with respect to a or b. Likewise, a comparison theorem and a Minkowski-type inequality involving the generalized Muirhead means $\Sigma_{a,b}$ are established.

MSC 2000. 26E60, 26D07.

Key words. Generalized Muirhead means, comparison of means, Minkowski's inequality.

REFERENCES

[1] Brenner, J.L. and Carlson, B.C., *Homogeneous mean values: weights and asymptotics*, J. Math. Anal. Appl., **123** (1987), 265–280.

[2] Bullen, P.S., Mitrinović, D.S. and Vasić, P.M., *Means and their Inequalities*, D. Reidel Publishing Co., Dordrecht, 1988.

[3] Czinder, P. and Páles, Zs., *A general Minkowski-type inequality for two variable Gini means*, Publ. Math. Debrecen, **57** (2000), 203–216.

[4] Czinder, P. and Páles, Zs., *Local monotonicity properties of two-variable Gini means and the comparison theorem revisited*, J. Math. Anal. Appl., **301** (2005), 427–438.

[5] Gini, C., *Di una formula comprensiva delle medie*, Metron, **13** (1938), 3–22.

[6] Hardy, G.H., Littlewood, J.E. and Pólya, G., *Inequalities*, Cambridge Univ. Press, Cambridge, 1934.

[7] Haruki, H. and Rassias, Th.M., *New characterizations of some mean-values*, J. Math. Anal. Appl., **202** (1996), 333–348.

[8] Leach, E.B. and Sholander, M.C., *Extended mean values*, Amer. Math. Monthly, **85** (1978), 84–90.

[9] Leach, E.B. and Sholander, M.C., *Multi-variable extended mean values*, J. Math. Anal. Appl., **104** (1984), 390–407.

[10] Losonczi, L. and Páles, Zs., *Minkowski's inequality for two variable Gini means*, Acta Sci. Math. (Szeged), **62** (1996), 413–425.

[11] Losonczi, L. and Páles, Zs., *Minkowski's inequality for two variable difference means*, Proc. Amer. Math. Soc., **126** (1998), 779–789.

[12] Neuman, E. and Sándor, J., *Inequalities involving Stolarsky and Gini means*, Math. Pannon., **14** (2003), 29–44.

[13] Páles, Zs., *Inequalities for sums of powers*, J. Math. Anal. Appl., **131** (1988), 265–270.

[14] Páles, Zs., *Inequalities for differences of powers*, J. Math. Anal. Appl., **131** (1988), 271–281.

[15] Pittenger, A.O., *The symmetric, logarithmic and power means*, Univ. Beograd. Publ. Elektrotehn. Fak., Ser. Mat. Fiz., No. 681 (1980), 19–23.
[16] Qi, F., *Generalized abstracted mean values*, JIPAM. J. Inequal. Pure Appl. Math., **1** (2000), Article 4 (electronic).
[17] Stolarsky, K.B., *Generalizations of the logarithmic mean*, Math. Mag., **48** (1975), 87–92.
[18] Trif, T., *Sharp inequalities involving the symmetric mean*, Math. Notes (Miskolc), **3** (2002), 157–164.

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Faculty of Mathematics and Computer Science
"Babeş-Bolyai" University
Str. M. Kogălniceanu nr. 1
400084 Cluj-Napoca, Romania
E-mail: ttrif@math.ubbcluj.ro