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EXPONENTIAL STABILITY AND EXPONENTIAL DICHOTOMY OF SEMIGROUPS OF LINEAR OPERATORS

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Abstract. The aim of this paper is to establish necessary and sufficient conditions for exponential stability of semigroups of linear operators and to show how this conditions can be applied in order to characterize the exponential dichotomy. First, we prove that an exponentially bounded semigroup is exponentially stable if and only if it is $(l^p(\mathbb{N}, X), l^\infty(\mathbb{N}, X))$-stable, where $p \in (1, \infty)$. After that this result is applied at the study of the exponential dichotomy of exponentially bounded semigroups. We deduce that an exponentially bounded semigroup is exponentially dichotomic if and only if the pair $(l^\infty(\mathbb{N}, X), l^p(\mathbb{N}, X))$ is admissible for it and an associated subspace is closed and complemented.

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Key words. Semigroup of linear operators, exponential stability, exponential dichotomy.

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