HYPERGEOMETRIC STARLIKE AND CONVEX FUNCTIONS WITH NEGATIVE COEFFICIENTS

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Abstract. In this paper we obtain several interesting properties of the hypergeometric function $F(a, b, c, d; e; z)$ where

$$F(a, b, c, d; e; z) = \sum_{n=0}^{\infty} \frac{(a, n)(b, n)(c, n)(d, n)}{(e, n)(1, n)} z^n.$$

In the class $H(a, b, c, d, e, z)$ of the hypergeometric functions $F(a, b, c, d; e; z)$ in the open unit disk $U = \{z \,||z| < 1\}$, we consider starlike and convex functions of order α with negative coefficients. These properties include conditions on a, b, c, d, e to guarantee $zF(a, b, c, d; e; z)$ to be in the subclasses of starlike and convex functions. We give also several interesting properties of the class $H(a, b, c, d; e; z)$.

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