

NEW CRITERIA FOR MEROMORPHIC P-VALENT CONVEX
FUNCTIONS

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Abstract. Let $G_n(\alpha)$ be the class of functions of the form $f(z) = \frac{a-p}{z^p} + \sum_{k=0}^{\infty} a_k z^k$ ($a-p \neq 0$, $p \in N = \{1, 2, \dots\}$) which are regular in the punctured disc $U^* = \{z : 0 < |z| < 1\}$ and satisfying $\operatorname{Re} \left\{ \frac{(D^{n+1}f(z))'}{(D^n f(z))'} - (p+1) \right\} < -\alpha$ ($n \in N_0 = \{0, 1, 2, \dots\}$, $|z| < 1$, $0 \leq \alpha < p$), where $D^n f(z) = \frac{a-p}{z^p} + \sum_{m=1}^{\infty} (p+m)^n a_{m-1} z^{m-1}$. It is proved that $G_{n+1}(\alpha) \subset G_n(\alpha)$. Since $G_0(\alpha)$ is the class of meromorphically p -valent convex functions of order α , $0 \leq \alpha < p$, all functions in $G_n(\alpha)$ are p -valent convex. A property preserving integrals is also considered.

MSC 2000. 30C45.

Key words. Regular, p -valent, convex, meromorphic.

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