**MATHEMATICA, Tome 48 (71), N° 1, 2006, pp. 41–48**

# LOEWNER CHAINS AND A MODIFICATION OF THE ROPER-SUFFRIDGE EXTENSION OPERATOR

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**Abstract.** In this paper we continue the study of the Roper-Suffridge extension operator. Let $f$ be a locally univalent function on the unit disc and let $Q : \mathbb{C}^{n-1} \to \mathbb{C}$ be a homogeneous polynomial of degree 2. We consider the family of operators extending $f$ to a holomorphic mapping from the unit ball $B^n$ in $\mathbb{C}^n$ into $\mathbb{C}^n$ given by $\Phi_{n,Q}(f)(z) = (f(z_1) + Q(\widetilde{z})f'(z_1), \widetilde{z}(f'(z_1))^{1/2})$, where $\widetilde{z} = (z_2, \ldots, z_n)$. This operator was recently introduced by Muir. In the case $Q \equiv 0$, this operator reduces to the well known Roper-Suffridge extension operator. We prove that if $f \in S$ then $\Phi_{n,Q}(f) \in S^0(B^n)$ whenever $\|Q\| \leq 1/4$. Our proof yields Muir's result that if $f \in S^*$ then $\Phi_{n,Q}(f)$ is also starlike on $B^n$. Moreover, if $f \in K$ is imbedded in a convex subordination chain $f(z_1, t)$ over $[0, \infty)$ then $\Phi_{n,Q}(f)$ is also imbedded in a c.s.c. over $[0, \infty)$ on $B^n$ whenever $\|Q\| \leq 1/2$.

**MSC 2000.** Primary: 32H. Secondary: 30C45.

**Key words.** Convexity, convex subordination chain, Loewner chain, parametric representation, Roper-Suffridge extension operator, starlikeness.

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Received November 26, 2005

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