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NONCONVEX MIXED QUASI VARIATIONAL INEQUALITIES

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Abstract. In this paper, we introduce a new class of mixed quasi variational inequalities, known as nonconvex mixed quasi variational inequalities in the setting of g-convexity. We suggest some algorithms for solving nonconvex mixed quasi variational inequalities by using the auxiliary principle technique. The convergence of the proposed methods either requires partially relaxed strongly monotonicity or pseudomononicity. We also introduce the concept of well-posedness for the nonconvex mixed quasi variational inequalities. As special cases, we obtain a number of known and new results for solving various classes of equilibrium and variational inequality problems. Our results can be considered as a significant improvement of the previously known results.

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Key words. Nonconvex functions, preinvex functions, variational-like inequalities, auxiliary principle, convergence analysis.

REFERENCES

[1] Baiocchi, C. and Capelo, A. *Variational and Quasi Variational Inequalities*, J. Wiley and Sons, New York (1994).

[2] Cottle, R.W., Giannessi, F. and Lions, J.L., *Variational Inequalities and Complementarity Problems: Theory and Applications*, J. Wiley and Sons, New York (1980).

[3] Cristescu, G. and Lupsa, L., *Non-Connected Convexities and Applications*, Kluwer Academic Publishers, Dordrecht, Holland (2002).

[4] Flores-Bazan, F., *Existence theorems for generalized noncoercive equilibrium problems: the quasi-convex case*, SIAM J. Optim., **11** (2000), 675–690.

[5] Giannessi, F. and Maugeri, A. *Variational Inequalities and Network Equilibrium Problems*, Plenum Press, New York (1995).

[6] Giannessi, F., Maugeri, A. and Pardalos, M. *Equilibrium Problems: Nonsmooth Optimization and Variational Inequality Models*, Kluwer Academic Publishers, Dordrecht, Holland (2001).

[7] Glowinski, R., Lions, J.L. and Trémolières, R., *Numerical Analysis of Variational Inequalities*, North-Holland, Amsterdam (1981).

[8] Goeleven, D. and Mantague, D. *Well-posed hemivariational inequalities*, Numer. Funct. Anal. Optim. **16** (1995), 909–921.

[9] Lions, J.L. and Stampacchia, G., *Variational inequalities*, Comm. Pure Appl. Math. **20** (1967), 493–512.

[10] Lucchetti, R. and Patrone, F., *A characterization of Tykhonov well-posedness for minimum problems with applications to variational inequalities*, Numer. Funct. Anal. Optim., **3** (1981), 461–478.

[11] Lucchetti, R. and Patrone, F., *Some properties of well-posed variational inequalities governed by linear operators*, Numer. Funct. Anal. Optim., **5** (1982-83), 349–361.

[12] Martinet, B., *Regularisation d'inequations variationelles par approximations successive*, Rev. d'Aut. Inform. Rech. Oper., **3** (1970), 154–159.

[13] MOSCO, U., *Implicit variational problems and quasi variational inequalities*, Lecture Notes in Mathematics, Springer-Verlag, Berlin, **543** (1976), 83–126.
[14] NANIEWICZ, Z. and PANAGIOTOPOULOS, P.D., *Mathematical Theory of Hemivariational Inequalities and Applications*, Marcel Dekker, New York (1995).
[15] NOOR, M.A., *New approximation schemes for general variational inequalities*, J. Math. Anal. Appl., **251** (2000), 217–229.
[16] NOOR, M.A., *Some developments in general variational inequalities*, Appl. Math. Computation, **x** (2004).
[17] NOOR, M.A., *Theory of general variational inequalities*, Preprint, Etisalat College of Engineering, Sharjah, UAE (2003).
[18] NOOR, M.A., *Iterative methods for general mixed quasi variational inequalities*, J. Optim. theory Appl., **119** (2003), 123–136.
[19] NOOR, M.A. and NOOR, K.I., *On general mixed quasi variational inequalities*, J. Optim. Theory Appl., **120** (2004), 579–599.
[20] NOOR, M.A., NOOR, K.I. and RASSIAS, TH.M., *Some aspects of variational inequalities*, J. Comput. Appl. Math., **47** (1993), 285–312.
[21] ROCKAFELLAR, R.T., *Monotone operators and the proximal point algorithms*, SIAM J. Control Optim., **14** (1976), 877–898.
[22] STAMPACCHIA, G., *Formes bilineaires coercivites sur les ensembles convexes*, Comptes Rend. Acad. Sciences, Paris, **258** (1964), 4413–4416.
[23] TSENG, P., *On linear convergence of iterative methods for variational inequality problem*, J. Comput. Appl. Math., **60** (1995), 237–252.
[24] YOUNESS, E.A., *E-convex sets, E-convex functions and E-convex programming*, J. Optim. Theory Appl., **102** (1999), 439–450.
[25] ZHU, D.L. and MARCOTTE, P., *Cocoercivity and its role in the convergence of iterative schemes for solving variational inequalities*, SIAM J. Optim., **6** (1996), 714–726.

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