

A SECOND-KIND INTEGRAL EQUATION METHOD FOR STOKES
FLOW PAST SMOOTH OBSTACLES IN A CHANNEL

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Abstract. In this paper we obtain a compound double-layer representation for Stokes flow due to the motion of a solid particle in an ambient flow located in a two-dimensional channel. Our indirect method is an extension of the well known Completed Double Layer Boundary Integral Equation Method of Power and Miranda [18] from the case of Stokes flow due to the motion of a solid particle in a viscous incompressible fluid of infinite expanse to the case of Stokes flow in a two-dimensional channel. The problem is reduced to the study of a system of Fredholm integral equations of the second kind. We prove that this system has a unique continuous solution. The numerical results are presented for Stokes flow due to the motion of a circular obstacle in a two-dimensional channel between two parallel solid walls. We also include some conclusions which refer to the effect of the walls on the considered Stokes flow.

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Key words. Stokes flow, completed double layer boundary integral equation method, Green's function, singularities of Stokes flow, boundary element method.

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