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EQUILIBRIUM PROBLEMS AND VARIATIONAL INEQUALITIES

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Abstract. In this paper, we suggest and analyze some iterative methods for solving equilibrium problems with trifunction by using the auxiliary principle technique. We prove that the convergence of the proposed methods either requires only pseudomonotonicity or partially relaxed strongly monotonicity. We also consider the concept of well-posedness for equilibrium problems with trifunction and obtain some new results. It is shown that the auxiliary principle technique developed in this paper can be extended for regularized equilibrium problems with some minor modifications. Since equilibrium problems with trifunction include the classical equilibrium problems, variational inequalities and complementarity problems as special cases, results proved in this paper continue to hold for these problems. Our results can be viewed as a novel application of the auxiliary principle technique.

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Key words. Equilibrium problems, variational inequalities, auxiliary principle, proximal methods, convergence, well-posedness.

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