SUBORDINATION CHAINS AND SOLUTIONS OF THE LOEWNER DIFFERENTIAL EQUATION IN $\mathbb{C}^n$

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Abstract. In this paper we continue the work begun in [8] and study the general solution of the Loewner differential equation on the unit ball in $\mathbb{C}^n$. We generalize to several variables a result of Becker concerning the form of arbitrary solutions to the Loewner differential equation. We do not require the solutions to be normalized. In particular, we determine the form of biholomorphic solutions, which need not be unique in higher dimensions. Also, we give some applications.

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Key words. Biholomorphic mapping, canonical solution, Loewner differential equation, Loewner chain, subordination, subordination chain.

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