

OPENNESS RESULTS FOR MULTIFUNCTIONS
VIA AN ABSTRACT SUBDIFFERENTIAL

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Abstract. In this paper we establish some openness-type results for set-valued maps in Banach spaces, using a coderivative which is defined via abstract subdifferentials. Mainly, results of openness at a linear rate are obtained. Then we apply these results to derive a necessary optimality condition for a nonlinear optimization problem, without any convexity assumptions.

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Key words. Abstract subdifferential, multifunction, openness at a linear rate, coderivative, optimization formula.

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