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ON A CERTAIN CLASS OF ANALYTIC FUNCTIONS WITH COMPLEX ORDER DEFINED BY SALAGEAN OPERATOR

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Abstract. We introduce a class, namely $R_\alpha^n(b,\beta)(b \neq 0$, complex, $0 < \beta \leq 1$, $n \in N_0 = \{0,1,2,\dots\}$ and $0 \leq \alpha < 1)$ of analytic functions defined by using Hadamard product $(D^n f * S_\alpha)(z)$ of the differential operator $D^n f(z) = z + \sum_{k=2}^{\infty} k^n a_k z^k$ and $S_\alpha(z) = \dfrac{z}{(1-z)^{2(1-\alpha)}}$ and satisfying the condition

$$\left| \frac{(D^n f * S_\alpha)'(z) - 1}{2\beta\left[(D^n f * S_\alpha)'(z) - 1 + b\right] - \left[(D^n f * S_\alpha)'(z) - 1\right]} \right| < 1, \quad z \in U.$$

In this paper we determine a sufficient condition, coefficient estimates, maximization of $\left|a_3 - \mu a_2^2\right|$ over the class $R_\alpha^n(b,\beta)$, distortion theorem and an argument theorem for the class $R_\alpha^n(b,\beta)$. Further we prove that some of the subclasses of $R_\alpha^n(b,\beta)$ are closed under convolution.

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