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IRREGULAR FORCED ALMOST PERIODIC SOLUTIONS OF ORDINARY LINEAR DIFFERENTIAL SYSTEMS

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Abstract. Let A be an almost periodic $(n \times n)$-matrix and let φ be an almost periodic vector. Suppose that $\operatorname{mod}(A) \cap \operatorname{mod}(\varphi) = \{0\}$. We say that the almost periodic solution x of the system

$$\dot{x} = A(t)x + \varphi(t), \quad t \in \mathbb{R}, \quad x \in \mathbb{R}^n,$$

is irregular with respect to $\operatorname{mod}(A)$ (or partially irregular) if $(\operatorname{mod}(x) + \operatorname{mod}(\varphi)) \cap \operatorname{mod}(A) = \{0\}$, and irregular forced if at the same time $\operatorname{mod}(x) \subseteq \operatorname{mod}(\varphi)$. We prove that an irregular with respect to $\operatorname{mod}(A)$ almost periodic solution is irregular forced in non-critical and some critical cases. The necessary and sufficient conditions for existence of irregular forced solutions are obtained.

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REFERENCES

[1] Barbălat, I. and Halanay, A., *Un probleme de vibrations non lineaires*, Colloq. Math., **18** (1967), 107–110.
[2] Corduneanu, C., *Almost Periodic Function*, Interscience Tracts Pure and Appl. Math., New York-London-Sydney-Toronto, 1969.
[3] Corduneanu, C., *Solutions presque-periodiques de certaines équatios paraboliques*, Mathematica (Cluj), **9** (1967), 241–244.
[4] Demenchuk, A.K., *On almost periodic solutions of ordinary differential systems*, Vestsi Akad. Navuk BSSR Ser. Fiz.-Mat. Navuk, **4** (1987), 16–22 (in Russian).
[5] Demenchuk, A.K., *Quasiperiodic solutions of differential systems with frequency bases of solutions and right sides that are linearly independent over* $\mathbb{Q}$, Differ. Uravn., **27** (1991), 1673–1679 (in Russian); English translation: Differ. Equ., **27** (1991), 1176–1181.
[6] Demenchuk, A.K., *On a class of quasiperiodic solutions of differential systems,* Dokl. Akad. Nauk Belarusi, **36** (1992), 14–17 (in Russian).
[7] Demenchuk, A.K., *Partially irregular almost periodic solutions of ordinary differential systems*, Math. Bohem., **126** (2001), 221–228.
[8] Demenchuk, A.K., *On partially irregular almost periodic solutions of differential systems with diagonal right-hand side*, Mem. Differential Equations Math. Phys., **23** (2001), 152–154.
[9] Demenchuk, A.K., *Quasiperiodic solutions of linear nonhomogeneous differential equations*, Differ. Uravn., **37** (2001), 730–734 (in Russian); English translation: Differ. Equ., **37** (2001), 763–767.
[10] Erugin, N.P., *Linear Systems of Ordinary Differential Equations with Periodic and Quasiperiodic Coefficients*, Izdatelstvo Akad. Nauk BSSR, Minsk, 1963 (in Russian).
[11] Fink, A.M., *Almost Periodic Differential Equations.* Lect. Notes in Math., **377**, Springer, Berlin, 1974.

[12] Grudo, E.I. and Demenchuk, A.K., *On periodic solutions with incommensurable periods of linear nonhomogeneous periodic differential systems*, Differ. Uravn., **23** (1987), 409–416 (in Russian); English translation: Differ. Equ., **23** (1987), 284–289.
[13] Halanay, A., *Soluţii aproape-periodice ale sistemelor de ecuaţii diferenţiale neliniare*, Comunicările Acad. R.P.R., **6** (1956), 13–17.
[14] Halanay, A., *Teoria Calitativă a Ecuaţilor Diferenţiale*, Editura Acad., Bucureşti, 1963.
[15] Kurzweil, J. and Veivoda, O., *On periodic and almost periodic solutions of the ordinary differential systems*, Czehoslovak Math. J., **5** (1955), 362–370.
[16] Levitan, B.M. and Zhikov, V.V., *Almost Periodic Functions and Differential Equations*, Izdatelstvo Moskovskogo Universiteta, Moskva, 1978 (in Russian).
[17] Malkin, I.G., *On resonance in quasiharmonic systems*, Prikl. Mat. Mekh., **18** (1954), 459–463 (in Russian).
[18] Massera, J.L., *Observationes sobre les soluciones periodicas de ecuaciones differentiales*, Boletin de la Facultad de Ingenieria Montevideo, **4** (1950), 37–45.
[19] Muszynska, A., Radziszewski, B., Szadkowski, J. and Ziemba, S., *Some problems of the theory of vibrations*, Nonlinear Vibration Problems, **13** (1972), 7–20.
[20] Samoilenko, A.M., *The Elements of Mathematical Theory of Multifrequency Oscillations*, Nauka, Moskva, 1987 (in Russian).

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