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MULTI-VALUED MAPPINGS ON METRIC SPACES

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Abstract. We consider a multi-valued mapping F of a complete metric space (X, d) into the class $B(X)$ of nonempty, bounded subsets of X. For A, B in $B(X)$ we define $\delta(A, B) = \sup\{d(a, b) : a \in A, b \in B\}$.

It is proved that if F satisfies the contractive type condition $\delta(Fx, Fy) \leq \max\{\varphi_1(d(x, y)), \varphi_2(\delta(x, Fx)), \varphi_3(\delta(y, Fy)), \varphi_4(\delta(x, Fy)), \varphi_5(\delta(y, Fx))\}$ for all $x, y \in X$, where $\varphi_j : [0, +\infty) \to [0, +\infty)$, $j \in \{1, 2, 3, 4, 5\}$, are real functions satisfying: (a) $\varphi_j(t) < t$ for $t > 0$, (b) $\lim_{s \to t+} \varphi_j(s) < t$ for $t > 0$, (c) φ_j are nondecreasing and (d) $\lim_{t \to +\infty} (t - \varphi_j(t)) = +\infty$, then there exists a unique point z in X such that $Fz = \{z\}$. This result is a generalization of known results in this area and include, as special cases some theorems of Fisher, Khan and Kubiaczyk, Reich, Ćirić and Rhoades and Watson.

Key words. Complete metric spaces, fixed points, multi-valued mappings.

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