P 0.1 Stabiliţi natura şi suma seriilor:

$$a)\ \sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right);\quad b)\ \sum_{n=1}^{\infty} \arctan\frac{1}{n^2+n+1};\quad c)\ \sum_{n=2}^{\infty} \ln\left(1-\frac{1}{n^2}\right);$$

$$d)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{2^{n-1}};\quad e)\ \sum_{n=1}^{\infty} \frac{2n-1}{2^n};\quad f)\ \sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)};$$

$$g)\ \sum_{n=1}^{\infty} \left(\sqrt{n+2}-2\sqrt{n+1}+\sqrt{n}\right);\quad h)\ \sum_{n=1}^{\infty} \frac{1}{\left(n+\sqrt{2}\right)\left(n+\sqrt{2}+1\right)};$$

$$i)\ \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)(n+2)};\quad j)\ \sum_{n=2}^{\infty} \frac{1}{n\ln n};\quad k)\ \sum_{n=3}^{\infty} \frac{1}{n(\ln n)\ln\ln n}.$$

P 0.2 Stabiliţi natura seriilor

$$a)\ \sum_{n=1}^{\infty} \frac{9+n}{2n+1};\quad b)\ \sum_{n=1}^{\infty} \frac{2^n+3^n}{2^{n+1}+3^{n+1}};\quad c)\ \sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+1}-\sqrt{2n-1}};$$

$$d)\ \sum_{n=1}^{\infty} \frac{n}{n+1};\quad e)\ \sum_{n=2}^{\infty} \frac{1}{\sqrt[n]{\ln n}};\quad f)\ \sum_{n=2}^{\infty} \frac{1}{\sqrt[n]{n}}.$$

P 0.3 Stabiliţi natura seriilor:

$$a)\ \sum_{n=1}^{\infty} \frac{1}{2n-1};\quad b)\ \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2};\quad c)\ \sum_{n=1}^{\infty} \frac{1}{\sqrt{4n^2-1}};$$

$$d)\ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}};\quad e)\ \sum_{n=1}^{\infty} \frac{n\cos^2(n\pi/3)}{2^n};\quad f)\ \sum_{n=2}^{\infty} \frac{\sqrt{n^2+n}}{\sqrt[3]{n^5-n}};$$

$$g)\ \sum_{n=1}^{\infty} \frac{1}{2^n-n+1};\quad h)\ \sum_{n=1}^{\infty} \frac{1}{3^n+n^2+n};\quad i)\ \sum_{n=1}^{\infty} \frac{\sqrt{7n}}{n^2+3n+5};$$

$$j)\ \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\sqrt[3]{n-1}};\quad k)\ \sum_{n=1}^{\infty} \frac{(\ln n)^{10}}{n^{1.1}};\quad l)\ \sum_{n=1}^{\infty} \frac{1}{1+\sqrt{2}+\sqrt[3]{3}+...+\sqrt[n]{n}};$$

$$m)\ \sum_{n=1}^{\infty} \left(2-\sqrt{e}\right)\left(2-\sqrt[3]{e}\right)...\left(2-\sqrt[n]{e}\right);\quad n)\ \sum_{n=1}^{\infty} \frac{e^n}{n(1+2^n)}$$

$$o)\ \sum_{n=1}^{\infty} \sin\frac{1}{n};\quad p)\ \sum_{n=1}^{\infty} \frac{\sqrt[4]{n^2-1}}{\sqrt{n^4-1}};\quad q)\ \sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2-1}}{\sqrt{n^3-1}}.$$

P 0.4 Stabiliţi natura seriilor:

$$a)\ \sum_{n=1}^{\infty}\frac{100^n}{n!};\quad b)\ \sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!};\quad c)\ \sum_{n=1}^{\infty}\frac{n!}{n^n};\quad d)\ \sum_{n=1}^{\infty}\frac{2^n\cdot n!}{n^n};\quad e)\ \sum_{n=1}^{\infty}\frac{3^n\cdot n!}{n^n};$$

$$f)\ \sum_{n=1}^{\infty}\frac{(n!)^2}{2^{n^2}};\quad g)\ \sum_{n=1}^{\infty}\frac{100\cdot 101\cdot ...\cdot(100+n)}{1\cdot 3\cdot ...\cdot(2n-1)};\quad h)\ \sum_{n=1}^{\infty}\frac{4\cdot 7\cdot ...\cdot(4+3n)}{2\cdot 6\cdot ...\cdot(2+4n)};$$

$$k)\ \sum_{n=1}^{\infty}\left(2-\sqrt[3]{2}\right)\left(2-\sqrt[5]{2}\right)...\left(2-\sqrt[2n+1]{2}\right);\quad l)\ \sum_{n=1}^{\infty}\frac{n^2}{\left(2+\frac{1}{n}\right)^n};$$

$$m)\ \sum_{n=1}^{\infty}\frac{1}{(2n^2+n+1)^{\frac{n+1}{2}}};\quad n)\ \sum_{n=1}^{\infty}\left(\frac{1^3+2^3+...+n^3}{n^3}-\frac{n}{4}\right)^n;$$

$$o)\ \sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+n^2+1}-\sqrt[3]{n^3-n^2+1}\right)^n;\quad p)\ \sum_{n=1}^{\infty}\frac{(3n)^2}{\sqrt{(16n^2+5n+1)^{n+1}}}.$$

P 0.5 Stabiliţi natura seriei cu termenul general

$$u_n=\frac{n^{n+1}}{\mathrm{e}^n\,(n+1)!}.$$

P 0.6 Pentru fiecare $a>0$, studiaţi natura seriei:

$$a)\ \sum_{n=1}^{\infty}\frac{1}{a^n+n};\quad b)\ \sum_{n=1}^{\infty}\frac{a^n}{\sqrt{n!}};\quad c)\ \sum_{n=1}^{\infty}a^{\ln n};\quad d)\ \sum_{n=1}^{\infty}\frac{a^n}{n^n};$$

$$e)\ \sum_{n=1}^{\infty}\left(\frac{n^2+n+1}{n^2}a\right)^n;\quad f)\ \sum_{n=1}^{\infty}\frac{3^n}{2^n+a^n};\quad g)\ \sum_{n=1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n}}{n^a}.$$

P 0.7 Pentru fiecare $a,b>0$, studiaţi natura seriei:

$$a)\ \sum_{n=1}^{\infty}\frac{a^n}{a^n+b^n};\quad b)\ \sum_{n=1}^{\infty}\frac{2^n}{a^n+b^n};\quad c)\ \sum_{n=1}^{\infty}\frac{a^nb^n}{a^n+b^n};$$

$$d)\ \sum_{n=1}^{\infty}\frac{(2a+1)\,(3a+1)\cdots(na+1)}{(2b+1)\,(3b+1)\cdots(nb+1)}.$$

P 0.8 Pentru fiecare $a,b\in\mathbb{R}$, $a>0$, stabiliţi natura seriei:

$$\sum_{n=1}^{\infty}\frac{(a+1)\,(a+2)\cdots(a+n)}{n!}\cdot\frac{1}{n^b}.$$

P 0.9 Fie $(u_n)_{n\geq 1}$ un şir de numere reale strict pozitive şi α un număr real. Pentru fiecare număr natural $n \geq 1$ punem

$$\Delta_n = \frac{u_{n+1}}{u_n} - \left(\frac{n}{n+1}\right)^{\alpha}.$$

Arătaţi că:

(i) Dacă $\alpha > 1$ şi există un număr natural n_0 cu proprietatea că

$$\Delta_n \leq 0, \text{ oricare ar fi } n \geq n_0,$$

atunci seria $\sum\limits_{n=1}^{\infty} u_n$ este convergentă.

(ii) Dacă $\alpha \leq 1$ şi există un număr natural n_0 cu proprietatea că

$$\Delta_n > 0, \text{ oricare ar fi } n \geq n_0,$$

atunci seria $\sum\limits_{n=1}^{\infty} u_n$ este divergentă.

P 0.10 Stabiliţi natura seriilor:

$$a) \sum_{n=1}^{\infty} \frac{(2n-1)!!}{(2n)!!} \frac{1}{2n+1}; \quad b) \sum_{n=1}^{\infty} \frac{(2n-1)!!}{(2n)!!}; \quad c) \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{n}{e}\right)^n.$$

P 0.11 Pentru fiecare $a > 0$, studiaţi natura seriilor:

$$a) \sum_{n=1}^{\infty} \frac{n!}{a(a+1)...(a+n)}; \quad b) \sum_{n=1}^{\infty} a^{-\left(1+\frac{1}{2}+...+\frac{1}{n}\right)}; \quad c) \sum_{n=1}^{\infty} \frac{a^n \cdot n!}{n^n}.$$

P 0.12 Dacă $\alpha, \beta, \gamma, x \in]0, +\infty[$, stabiliţi natura seriei

$$\sum_{n=1}^{\infty} \frac{\alpha(\alpha+1)...(\alpha+n-1)\,\beta(\beta+1)...(\beta+n-1)}{\gamma(\gamma+1)...(\gamma+n-1)} x^n$$

(numită seria hipergeometrică a lui Gauss).

P 0.13 Să se stabilească natura seriilor:

$$a) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n+1)^{n+1}}{n^{n+2}}; \quad b) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n+1)^{n-1}}{n^{n+1}};$$

$$c) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n+1}{3^n}; \quad d) \sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{\ln n}; \quad e) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n(n+1)};$$

$$f) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n(n+1)}}; \quad g) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n+\sqrt{5}};$$

$$h)\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n-1)!!}{(2n)!!}.$$

P 0.14 Să se stabilească natura seriilor:

$$a)\sum_{n=1}^{\infty}(-1)^{\frac{n(n+1)}{2}}\frac{n^{100}}{2^n};\quad b)\sum_{n=1}^{\infty}(-1)^{\frac{n(n+1)}{2}}\sin\frac{\pi}{n\sqrt{n+1}}.$$

P 0.15 Determinaţi valorile lui $x\in\mathbb{R}$ pentru care seriile următoare sunt convergente:

$$a)\sum_{n=1}^{\infty}\frac{1}{(n+|x|)^2};\quad b)\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{2n(2n-1)};$$

$$c)\sum_{n=1}^{\infty}\frac{x}{(1+nx^2)\sqrt{n}};\quad d)\sum_{n=1}^{\infty}\frac{x}{1+|x|\sqrt{n}}.$$

P 0.16 Pentru fiecare $a\in\mathbb{R}$, stabiliţi natura seriilor;

$$a)\sum_{n=1}^{\infty}\cos(na)\sin\frac{a}{n};\quad b)\sum_{n=1}^{\infty}\sin(na)\sin\frac{a}{n};$$

$$c)\sum_{n=1}^{\infty}\cos(na)\tan\frac{a^2}{n};\quad d)\sum_{n=1}^{\infty}\sin(na)\tan\frac{a}{n};$$

$$e)\sum_{n=1}^{\infty}(-1)^{\left[\frac{n}{4}\right]}\ln\frac{n+a^2}{n}.$$

P 0.17 Să se arate că pentru orice număr natural p are loc inegalitatea

$$\sum_{n=1}^{\infty}\frac{1}{(n+1)\sqrt[p]{n}}<p.$$