Knowledge Discovery with FCA

Lecture 3: Order Theory in a Nutshell

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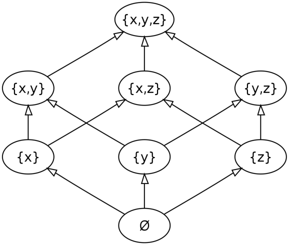


Relations

- What is a relation?
- How can we formalize it?
- What types of relations do we know?

Relations

What is a relation?

`HasFriends`

`IsRelatedTo`

`Division`

`GreaterThan`

`Any Prolog predicate`

Relations

How can we formalize it?

Relations

How can we formalize it?

Definition

Let A and B two sets. A subset $R \subseteq A \times B$ is called binary relation between A and B.

Ternary relations
Quaternary relations
n-ary relations

Relations

What types of relations do we know?

RELATIONS

What types of relations do we know?

1. Classification

Relations

What types of relations do we know?

1. Classification Equivalence relations
2. Hierarchies, ordering

RELATIONS

What types of relations do we know?

1. Classification Equivalence relations
2. Hierarchies, ordering Order relations
3. Equality relation

Relations

How can we visualize relations?

Relations: Properties

Definition

Let A be a set, $R \subseteq A \times A$ a binary relation on A. The relation R is called

1. *reflexive if for all $a \in A$, $(a, a) \in R$,*
2. *symmetric if for all $a, b \in A$, if $(a, b) \in R$ then $(b, a) \in R$,*
3. *antisymmetric if $(a, b) \in R$ and $(b, a) \in R$ then $a = b$*
4. *transitive if for every $(a, b), (b, c) \in R$ we have $(a, c) \in R$*

Relation: Properties

- How can we visualize these properties?

Relation: Properties

- How can we visualize these properties?
- Can you give some examples?
- Discussions in the group.

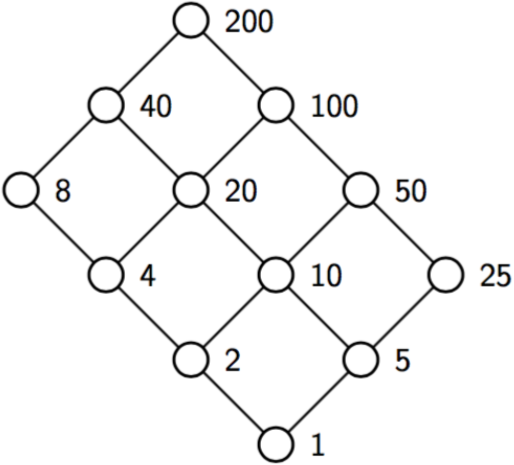
Order relations

Definition

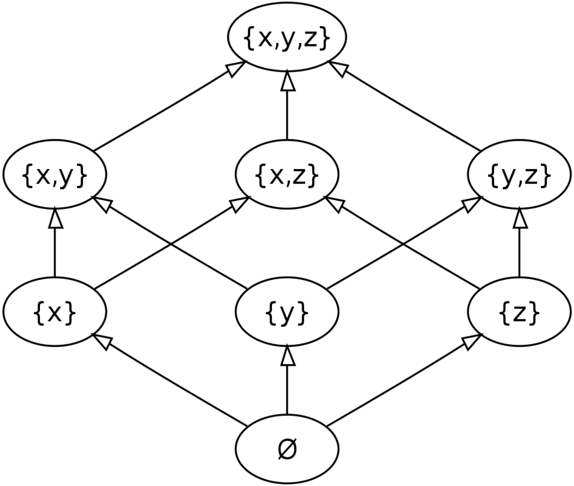
A relation $R \subseteq A \times A$ is called order relation, if R is reflexive, transitive and antisymmetric.

- Can you give examples of order relations?
- Is there any possibility to represent them graphical?

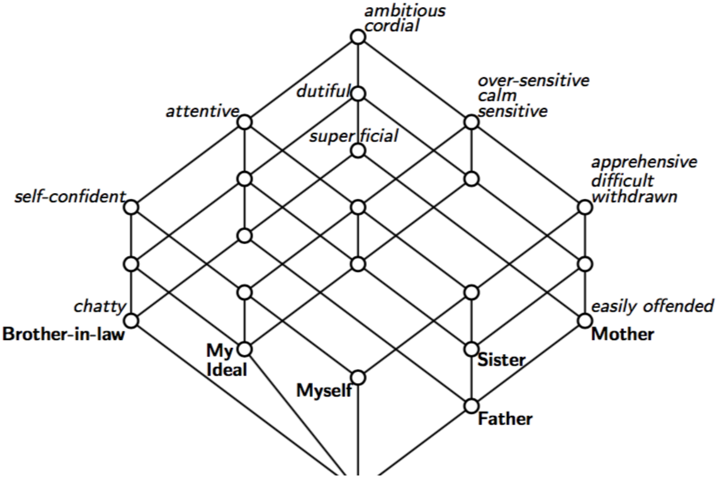
Order diagram



ORDER DIAGRAM



ORDER DIAGRAM



Special elements in an order relation

1. upper neighbour. Examples?

Special elements in an order relation

1. upper neighbour. Examples?
2. lower neighbour. Examples?

SPECIAL ELEMENTS IN AN ORDER RELATION

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3. least upper neighbour, **supremum**. Examples?

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6. smallest element **0**. Examples?

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3. least upper neighbour, **supremum**. Examples?
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5. largest element **1**. Examples?
6. smallest element **0**. Examples?
7. atom, coatom, irreducible. Examples?

Examples

- $(\mathbb{N}, \leq)$. The order diagram is ....

EXAMPLES

- $(\mathbb{N}, \leq)$. The order diagram is ....
- $(\mathbb{N}, |)$. The order diagram is ....

EXAMPLES

- $(\mathbb{N}, \leq)$. The order diagram is ....
- $(\mathbb{N}, |)$. The order diagram is ....
- Boolean lattice. The order diagram is ....

LATTICE

Definition

Let $(L, \leq)$ be an ordered set. Then L is called lattice if for every two elements $a, b \in L$ there exists the infimum and supremum of a and b, denoted by $\inf(a, b) = a \wedge b$ and $\sup(a, b) = a \vee b$, respectively. The lattice L is called complete if for every subset $A \subseteq L$, there exists the infimum and supremum of A.

Discussion about lattices, complete lattices, largest, smallest element, $\inf \emptyset$, $\sup \emptyset$.