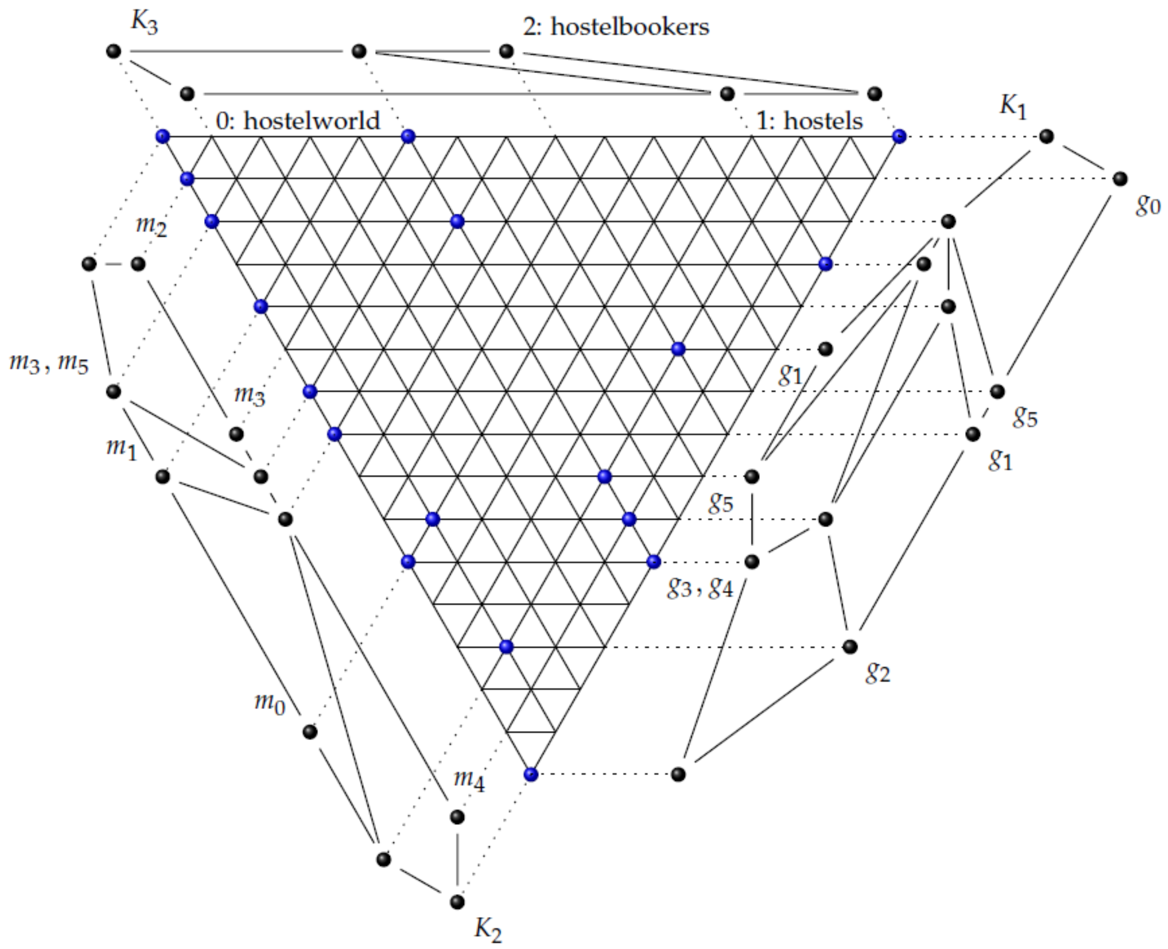
Knowledge Discovery

Lecture 7: Triadic FCA

Triadic FCA (3FCA)

- Target: triadic data sets either having a natural 3D structure or mapped on 3D in order to find relevant knowledge structures.
- Mine after relevant knowledge
- Cluster relevant information into triconcepts
- Represent triconcepts in a form which is suitable for decision making
- Analyse the structure of the triconcepts set in order to find more relevant knowledge structures.

Trilattice example (4)



Source: Factor Analysis of triadic data - C.V.Glodeanu

Disadvantages of the trilattice representation

- no automated tool to generate it
- difficult to read and to navigate in
- the underlying structure of the triconcepts cannot be read from the trilattice

Proposed solution

- A local navigation paradigm in the tricontexts:
- one can get from a triconcept to another if it is **reachable**
- once chosen a triconcept you can see all possible next steps

Derived contexts

Definition (Derived contexts)

Every triadic context (K_1, K_2, K_3, Y) gives rise to the following projected dyadic contexts:
$\mathbb{K}^{(1)} := (K_1, K_2 \times K_3, Y^{(1)})$ *with* $gY^{(1)}(m,b) :\Leftrightarrow (g,m,b) \in Y$,
$\mathbb{K}^{(2)} := (K_2, K_1 \times K_3, Y^{(2)})$ *with* $mY^{(2)}(g,b) :\Leftrightarrow (g,m,b) \in Y$,
$\mathbb{K}^{(3)} := (K_3, K_1 \times K_2, Y^{(3)})$ *with* $bY^{(3)}(g,m) :\Leftrightarrow (g,m,b) \in Y$.
For $\{i,j,k\} = \{1,2,3\}$ *and* $A_k \subseteq K_k$, *we define* $\mathbb{K}^{(ij)}_{A_k} := (K_i, K_j, Y^{(ij)}_{A_k})$, *where* $(a_i, a_j) \in Y^{(ij)}_{A_k}$ *if and only if* $(a_i, a_j, a_k) \in Y$ *for all* $a_k \in A_k$.

- $\mathbb{K}^{(i)}$ = **flattened** versions of the triadic context, obtained by putting the "slices" of (K_1, K_2, K_3, Y) side by side.
- $\mathbb{K}^{(ij)}_{A_k}$ = the intersection of all those slices that correspond to elements of A_k.

Derivation operators

Definition ((i)-derivation operators)

For $\{i, j, k\} = \{1, 2, 3\}$ *with* $j < k$ *and for* $X \subseteq K_i$ *and* $Z \subseteq K_j \times K_k$ *the* (i)*-derivation operators are defined by:*

$X \mapsto X^{(i)} := \{(a_j, a_k) \in K_j \times K_k \mid (a_i, a_j, a_k) \in Y \text{ for all } a_i \in X\}.$

$Z \mapsto Z^{(i)} := \{a_i \in K_i \mid (a_i, a_j, a_k) \in Y \text{ for all } (a_j, a_k) \in Z\}.$

Definition ((i, j, X_k)-derivation operators)

For $\{i, j, k\} = \{1, 2, 3\}$ *and* $X_i \subseteq K_i, X_j \subseteq K_j, X_k \subseteq K_k$*, the* (i, j, X_k)*-derivation operators are defined by*

$X_i \mapsto X_i^{(i,j,X_k)} := \{a_j \in K_j \mid (a_i, a_j, a_k) \in Y \text{ for all } (a_i, a_k) \in X_i \times X_k\}$

$X_j \mapsto X_j^{(i,j,X_k)} := \{a_i \in K_i \mid (a_i, a_j, a_k) \in Y \text{ for all } (a_j, a_k) \in X_i \times X_k\}.$

Recall triadic concepts

- triadic concepts are maximal cuboids of incidences

Definition

A **triadic concept** *(short:* **triconcept***) of* $\mathbb{K} := (K_1, K_2, K_3, Y)$ *is a triple* (A_1, A_2, A_3) *with* $A_i \subseteq K_i$ *for* $i \in \{1, 2, 3\}$ *and* $A_i = (A_j \times A_k)^{(i)}$ *for every* $\{i, j, k\} = \{1, 2, 3\}$ *with* $j < k$*. The sets* A_1, A_2*, and* A_3 *are called* **extent, intent,** *and* **modus** *of the triadic concept, respectively. We let* $\mathfrak{T}(\mathbb{K})$ *denote the set of all triadic concepts of* $\mathbb{K}$*.*

Exercise assignement

Let **K**=(K1, K2, K3, Y) be a triadic context, where

K1 = {1,2,3,4},

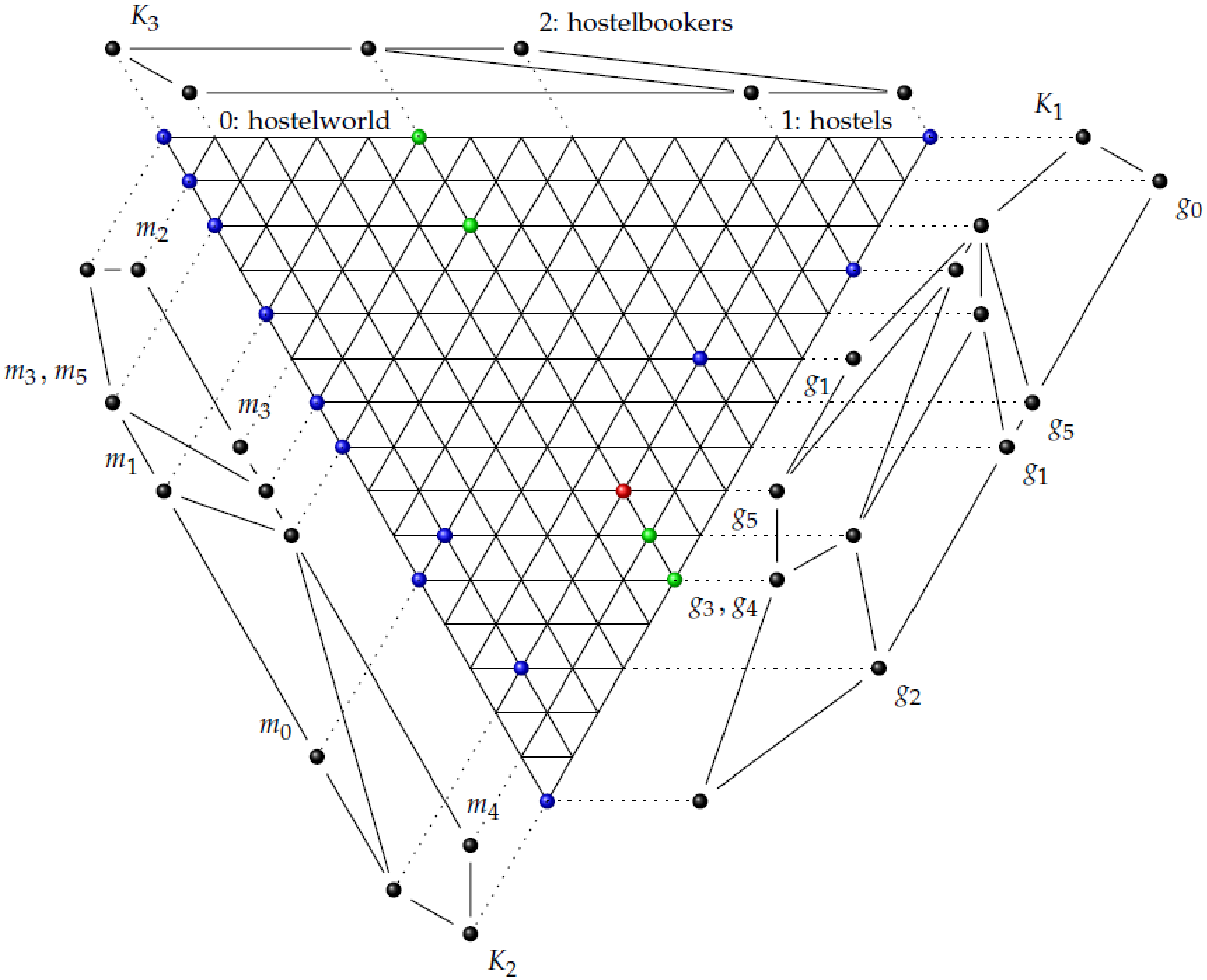
K2 = {a,b,c,d},

K3 = {α, β, γ}, and

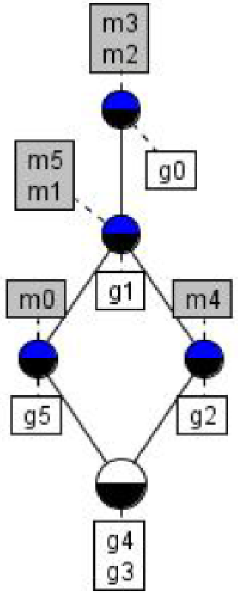
Y={(1,a,β), (1,a,γ), (1,b,α), (1,b,β), (1,c,α), (1,c,β), (2,b,α), (2,b,β), (2,a,β),(2,a,γ),(3,a,β),(3,a,γ),(3,d,α),(4,d,β),(4,a,β),(4,a,γ),(4,b,α), (4,b,β)

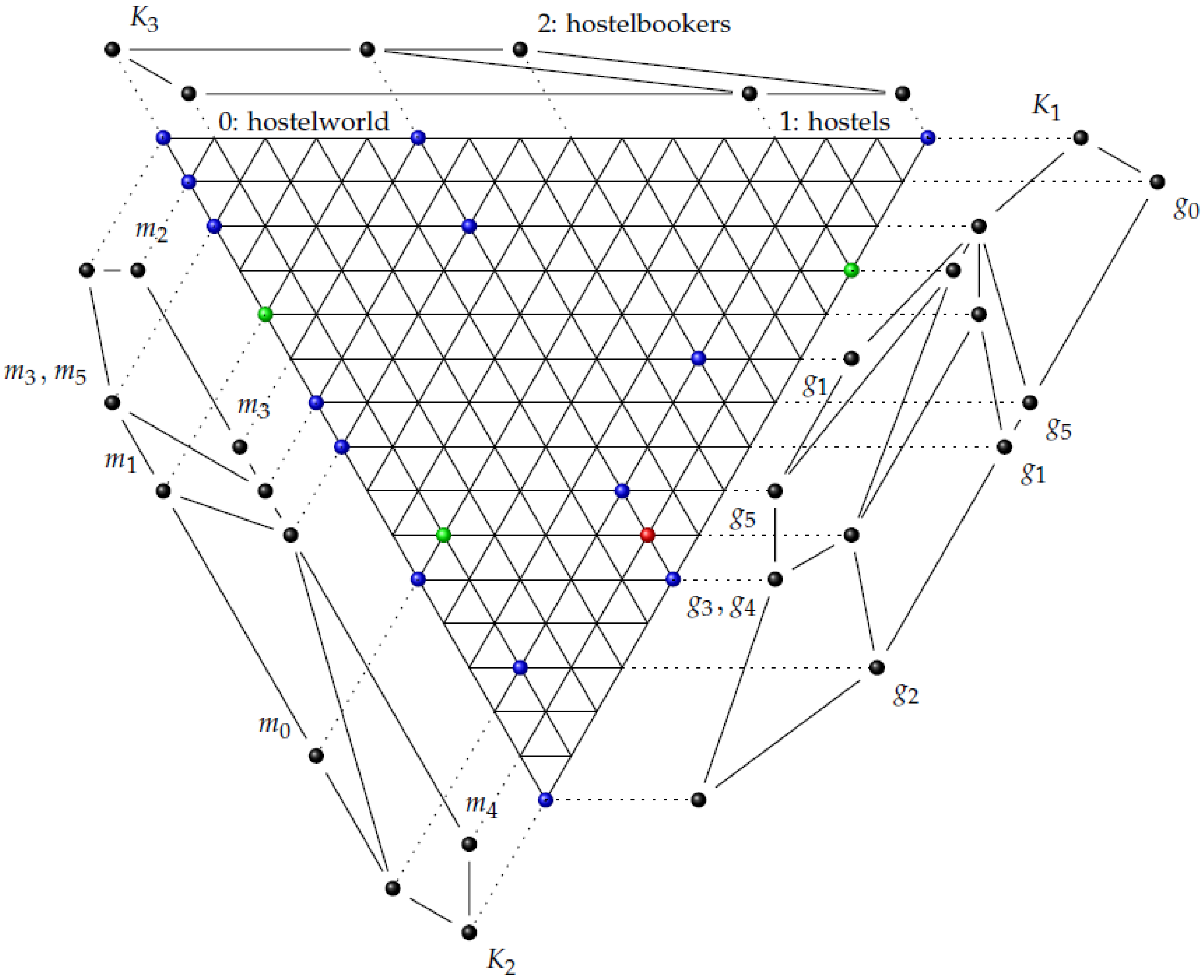
1. Give all three slice representations of **K**.
2. Compute three different triconcepts of **K**.
3. For a triconcept (A1, A2, A3) compute the projections $\mathbf{K}^{ij}_{Ak}$.

Navigation example



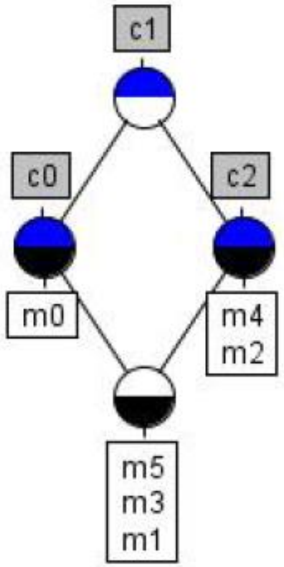
Choose $T := (\{g_3, g_4, g_5\}, \{m_0, m_1, m_2, m_3, m_5\}, \{c_1, c_2\})$ as the triconcept wherefrom local navigation starts and consider perspective 3 (i.e., modus). By projecting along $\{c_1, c_2\}$, we obtain the following concept lattice.
Triconcept T corresponds to the leftmost dyadic concept.





K_3
2: hostelbookers
0: hostelworld
1: hostels
K_1
g_0
m_2
m_3, m_5
m_3
g_1
g_5
m_1
g_1
g_5
g_3, g_4
g_2
m_0
m_4
K_2

By choosing the rightmost concept of this lattice corresponding to the triconcept $(\{g_2, g_3, g_4\}, \{m_2, m_3, m_4\}, \{c_1, c_2\})$ as a next step and the first perspective the reachable triconcepts are:



Reachable triconcepts

Definition

For (A_1, A_2, A_3) *and* (B_1, B_2, B_3) *triadic concepts, we say that* (B_1, B_2, B_3) *is* directly reachable *from* (A_1, A_2, A_3) using perspective (1) *and we write* $(A_1, A_2, A_3) \prec_1 (B_1, B_2, B_3)$ *if and only if* $(B_2, B_3) \in \mathfrak{B}(\mathbb{K}^{(23)}_{A_1})$*. Analogously, we can define direct reachability using perspectives* (2) *and* (3)*.*
We say that (B_1, B_2, B_3) *is* directly reachable *from* (A_1, A_2, A_3) *if it is directly reachable using at least one of the three perspectives, that is, formally*
$(A_1, A_2, A_3) \prec (B_1, B_2, B_3) :\Leftrightarrow [(A_1, A_2, A_3) \prec_1 (B_1, B_2, B_3)] \vee$
$[(A_1, A_2, A_3) \prec_2 (B_1, B_2, B_3)] \vee [(A_1, A_2, A_3) \prec_3 (B_1, B_2, B_3)]$.

Remark

$\mathbb{K}^{(ij)}_{A_k} := (K_i, K_j, Y^{(ij)}_{A_k})$*, where* $(a_i, a_j) \in Y^{(ij)}_{A_k}$ *if and only if* $(a_i, a_j, a_k) \in Y$ *for all* $a_k \in A_k$.

Mutually reachable triconcepts

Proposition

Let $(A_1, A_2, A_3), (B_1, B_2, B_3)$ be two triconcepts. If $A_i = B_i$ for an $i \in \{1, 2, 3\}$ then $(A_1, A_2, A_3) \prec_i (B_1, B_2, B_3)$ and $(B_1, B_2, B_3) \prec_i (A_1, A_2, A_3)$.

Reachability for triconcepts

Definition

We define the reachability relation *between two triconcepts as being the transitive closure of the direct reachability relation $\prec$. We denote this relation by $\lhd$.*

Definition

The equivalence class of a triconcept (A_1, A_2, A_3) with respect to the preorder $\lhd$ on $\mathfrak{T}(\mathbb{K})$ will be called a reachability cluster *and denoted by $[(A_1, A_2, A_3)]$.*

Obtaining the clusters of triconcepts using a graph

- Consider a directed graph with triconcepts as nodes and with directed edges between the triconcepts for which the reachability relation holds
- The strongly connected components of the graph represent the **clusters** of triconcepts.

More than two clusters

α	1	2	3
a	$\times$		
b			
c			

β	1	2	3
a	$\times$		
b		$\times$	
c			

γ	1	2	3
a	$\times$		
b		$\times$	
c			$\times$

The triconcepts are partitioned in clusters the following way:

$C_1 = \{(\{c\}, \{3\}, \{\gamma\})\}$

$C_2 = \{(\{b\}, \{2\}, \{\beta, \gamma\})\}$

$C_3 = \{(\{a\}, \{1\}, \{\alpha, \beta, \gamma\}), (\{a, b, c\}, \{1, 2, 3\}, \emptyset),$
$(\{a, b, c\}, \emptyset, \{\alpha, \beta, \gamma\}), (\emptyset, \{1, 2, 3\}, \{\alpha, \beta, \gamma\})\}$

$\mathbf{C_1 < C_2 < C_3}$

Remark

The triconcepts $(\{c\}, \{3\}, \{\gamma\})$ *and* $(\{b\}, \{2\}, \{\beta, \gamma\})$ *have disjoint extents and intents, but* $(\{c\}, \{3\}, \{\gamma\}) \prec_3 (\{b\}, \{2\}, \{\beta, \gamma\})$.

Reachability in composed tricontexts

There is a way of composing several tricontexts such that the reachability clusters of the composed tricontext coincide with the union of the reachability clusters of the constituents, except for the greatest cluster.

Definition

Given tricontexts $\mathbb{K}_1 := (K_1^1, K_2^1, K_3^1, Y^1), \ldots, \mathbb{K}_n := (K_1^n, K_2^n, K_3^n, Y^n)$, *with* K_i^j *and* K_i^k *being disjoint for all* $j \neq k$ *and all* $i \in \{1, 2, 3\}$, *their* composition $\mathbb{K}_1 \uplus \ldots \uplus \mathbb{K}_n$ *is the tricontext* $\mathbb{K} := (K_1, K_2, K_3, Y)$ *with* $K_i := \bigcup_{k=1}^{n} K_i^k$ *and* $Y := \bigcup_{k=1}^{n} Y^k$.

Size of the tricontext vs. Number of reachability clusters

We can find qubic tricontexts (i.e., $|K_1| = |K_2| = |K_3| = n$), where the number of clusters equals $n + 1$:

Proposition

Let $\mathbb{K} = (K_1, K_2, K_3, Y)$ be a tricontext of size $n \times n \times n$ with $K_1 = \{k_i^1 \mid 1 \leq i \leq n\}$, $K_2 = \{k_i^2 \mid 1 \leq i \leq n\}$, $K_3 = \{k_i^3 \mid 1 \leq i \leq n\}$. Let the relation Y be the spatial main diagonal of the tricontext, meaning that a triple $(k_i^1, k_j^2, k_l^3) \in Y$ iff $i = j = k$. Then there are n+1 clusters, n minimal clusters and the maximal cluster.