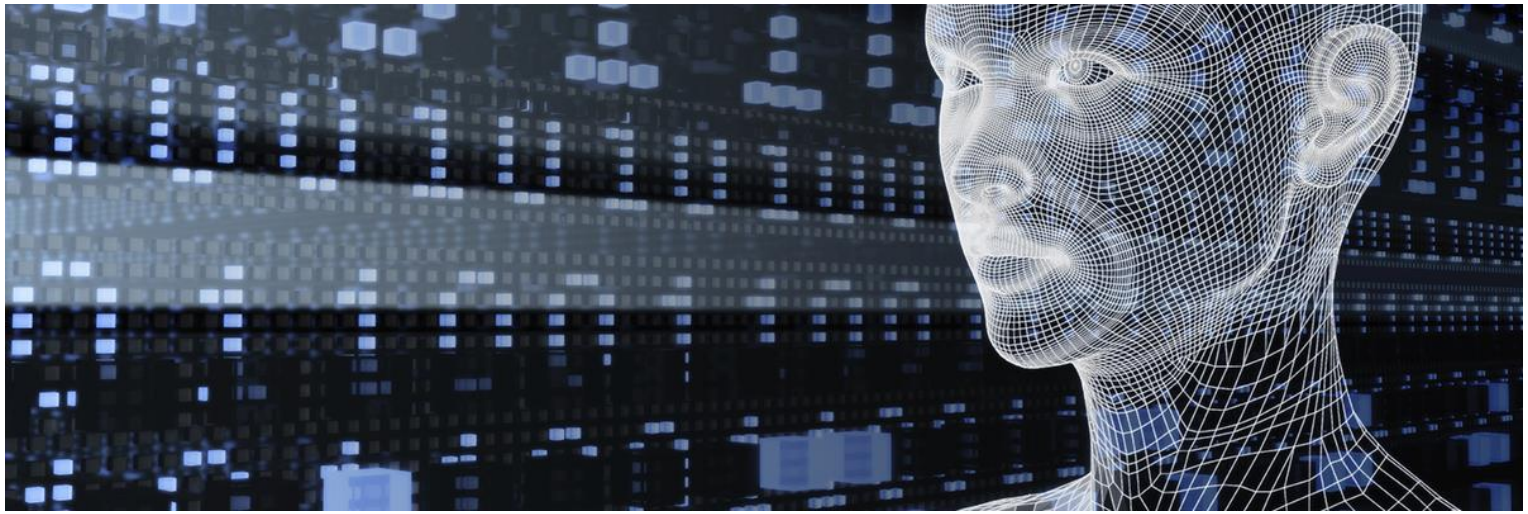
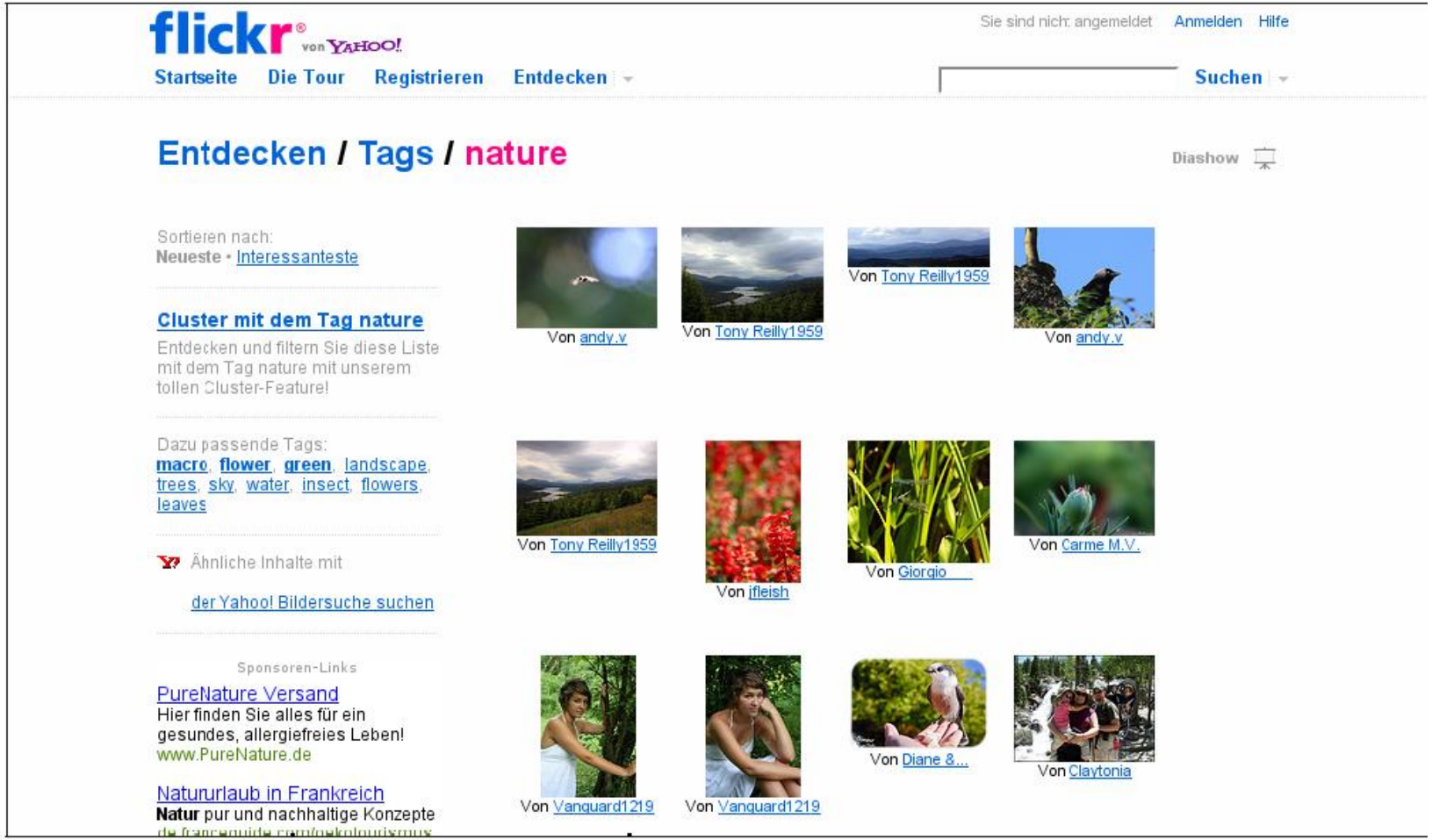
Knowledge Discovery

Lecture 6: Triadic FCA



Motivation: Collaborative Tagging Systems



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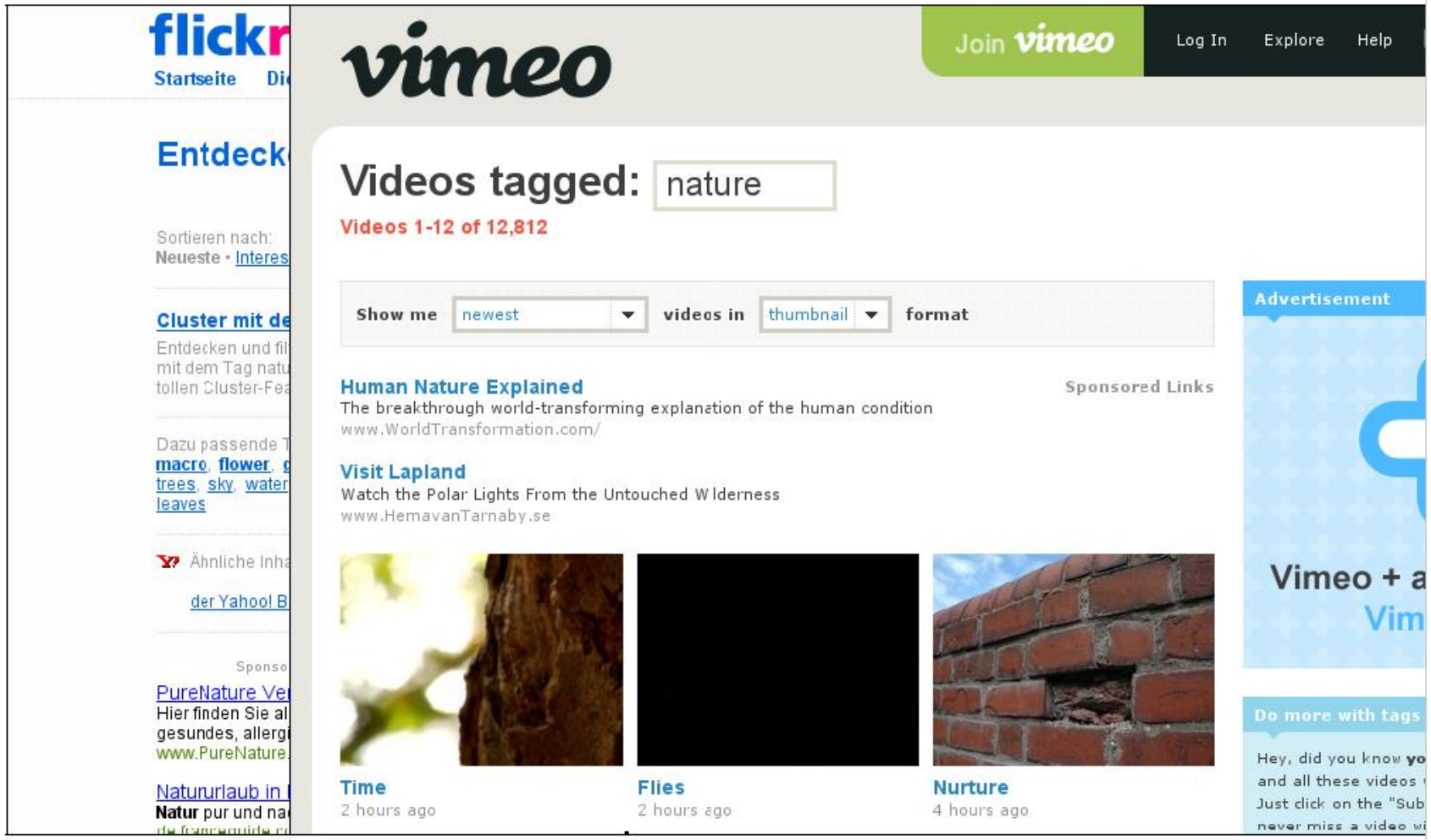

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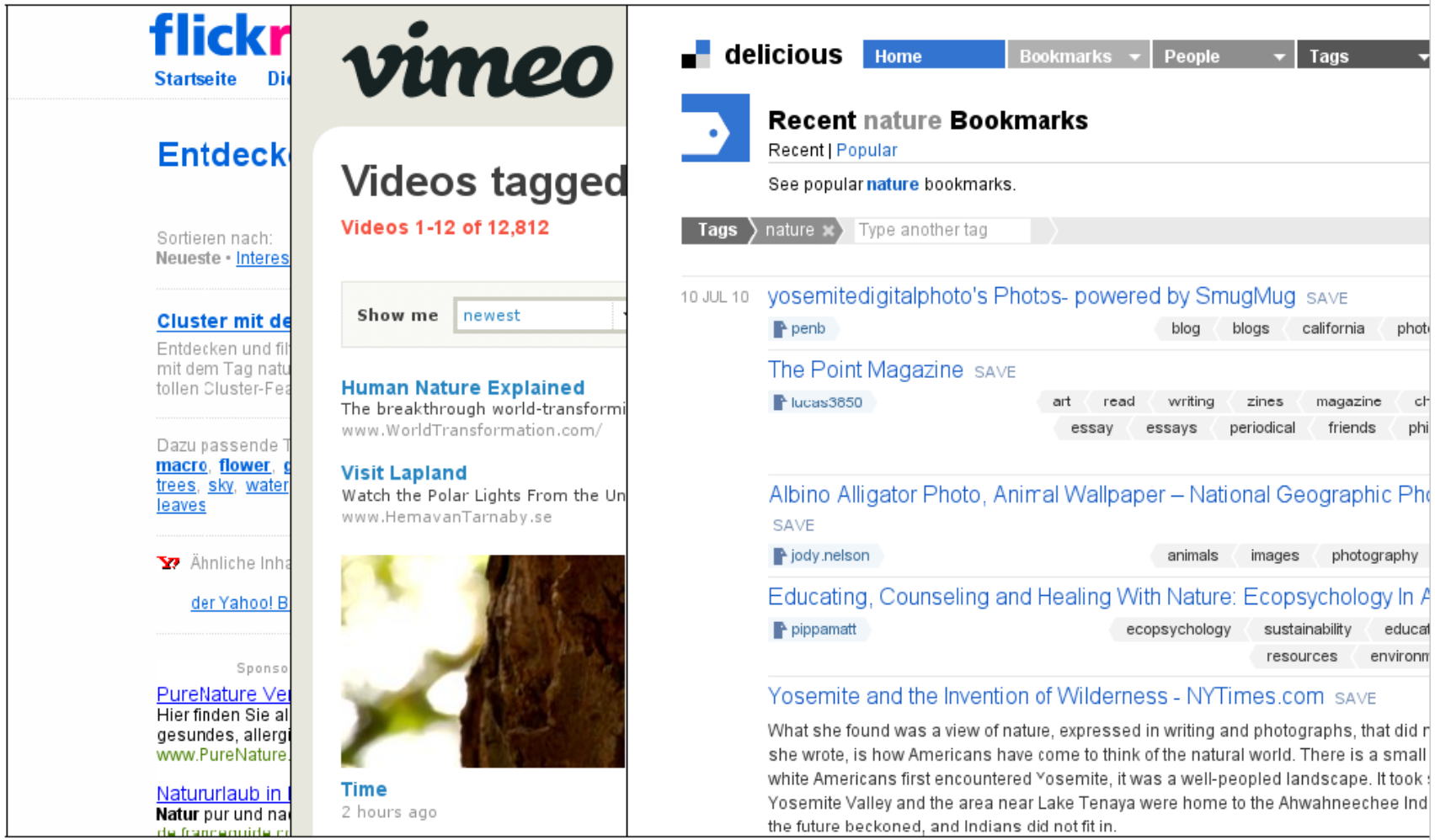

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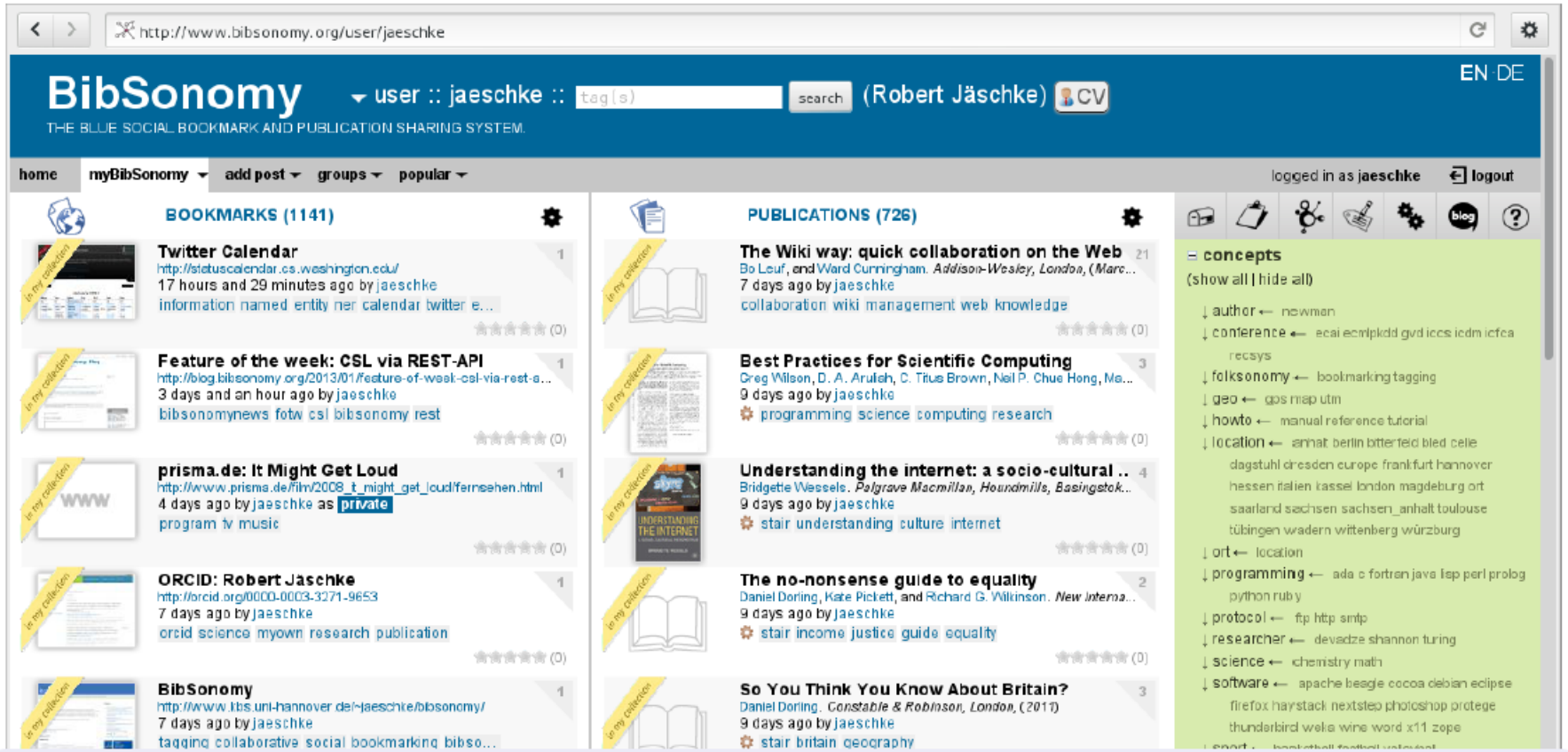
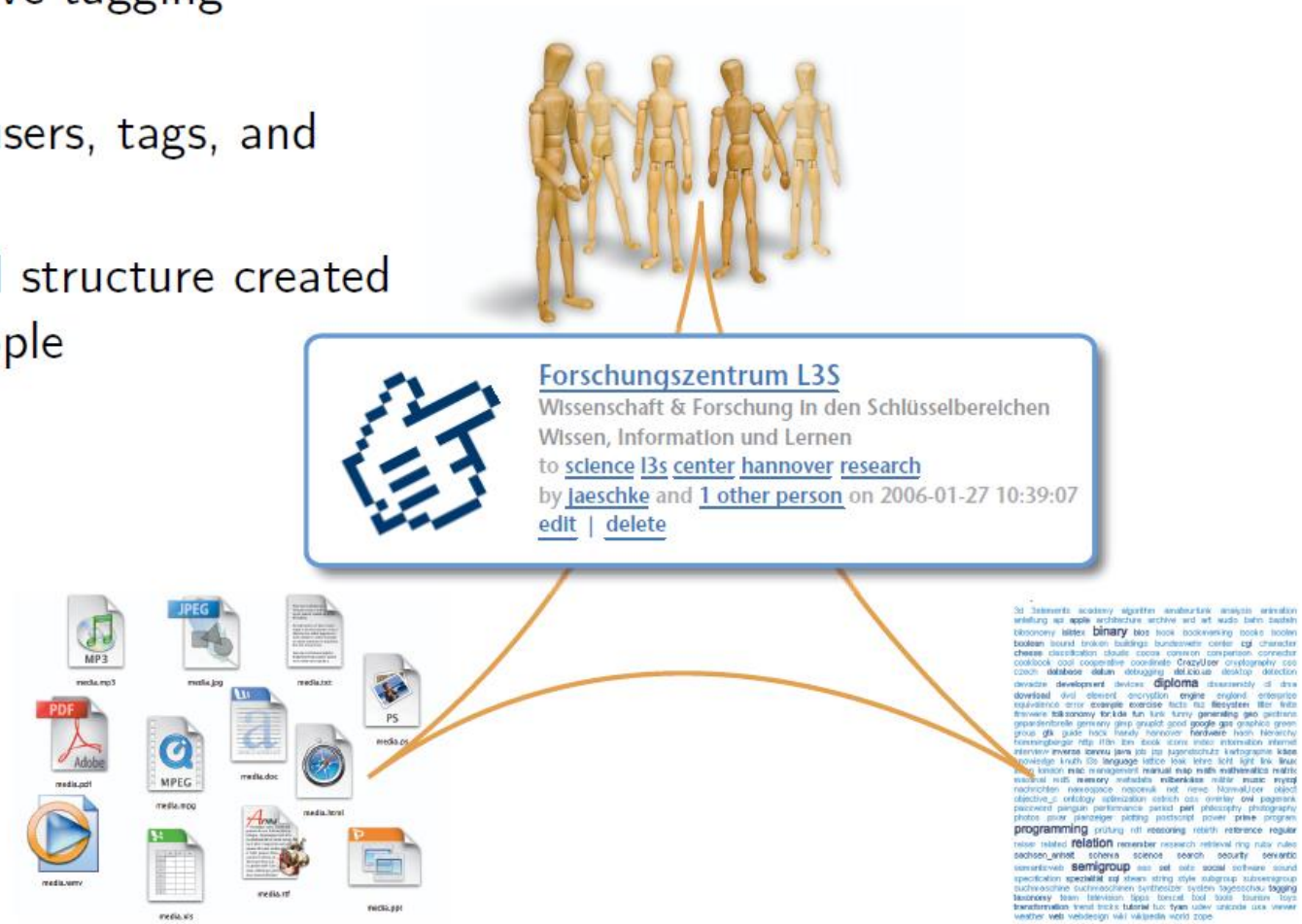


- manage your web bookmarks and publication references
- open for the public since beginning of 2006, $>$ 5 000 active users
- developed and operated at L3S Research Center

Folksonomies

- data structure of collaborative tagging systems
- connects users, tags, and resources
- conceptual structure created by the people



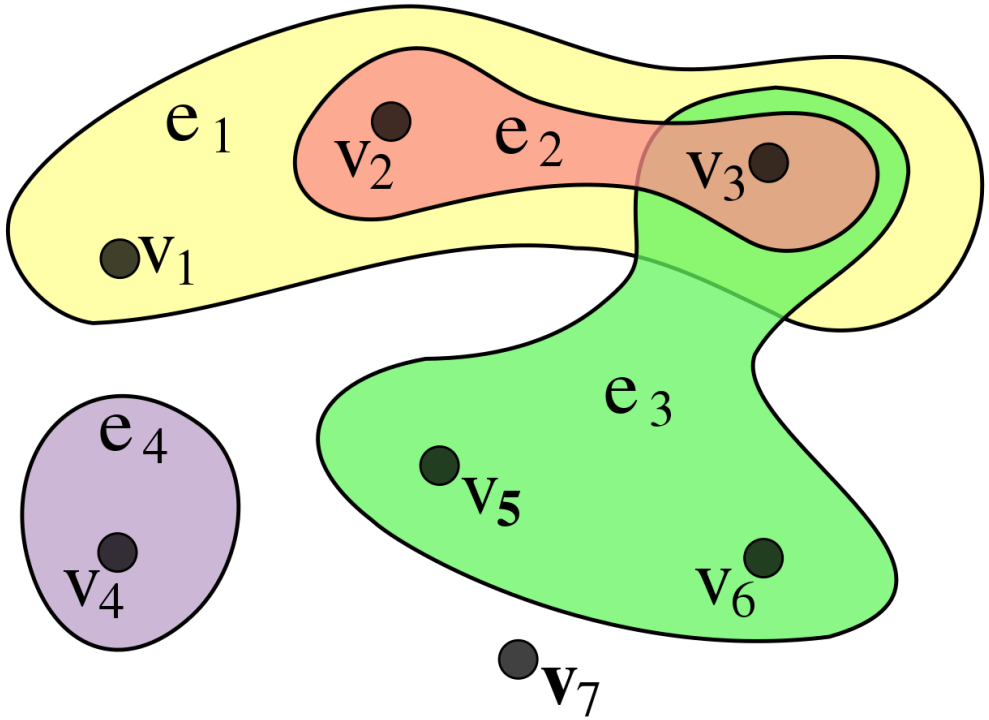
Folksonomies: Hypergraph

- What is a hypergraph?

Hypergraph

- Generalization of a graph in which an edge can join any number of vertices
- H=(X,E).
- X = set of vertices;
- E subset of P(X) (but not empty)

Hypergraph

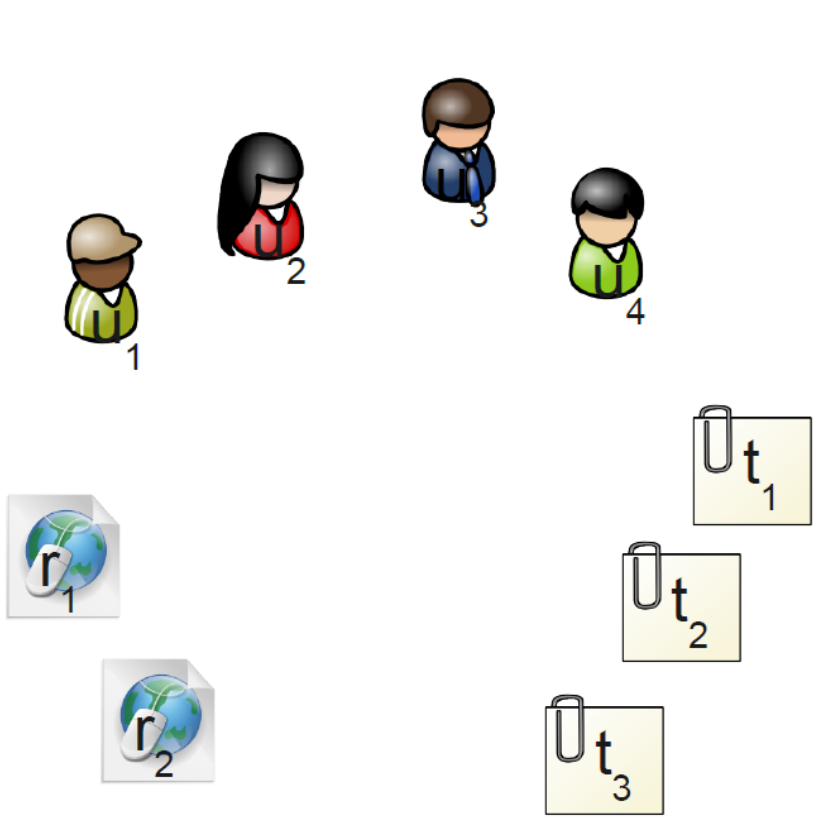


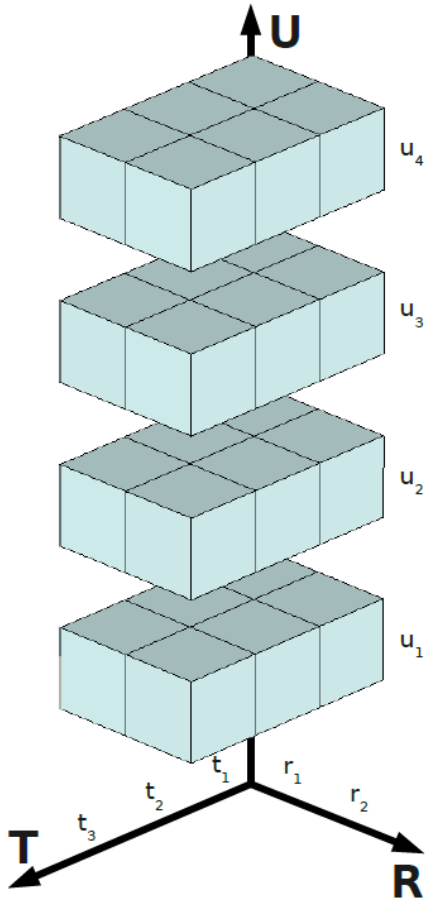
An example of a hypergraph, with $X = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and $E = \{e_1, e_2, e_3, e_4\} = \{\{v_1, v_2, v_3\}, \{v_2, v_3\}, \{v_3, v_5, v_6\}, \{v_4\}\}$. Here, edges do not just connect two vertices but several, and are represented by colors.

Hypergraph applications

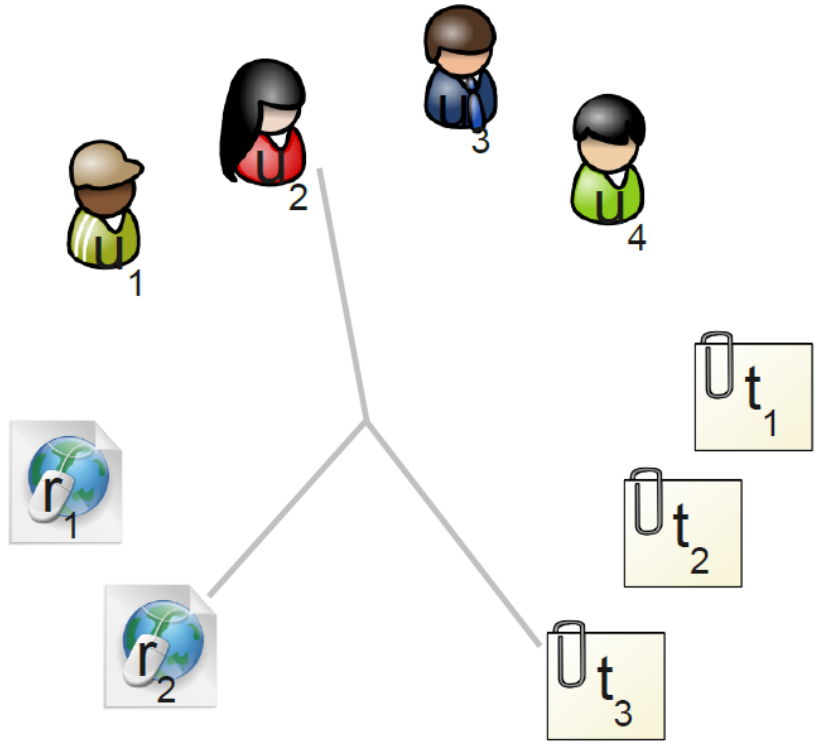
- Machine learning
- Recommender systems: communities as hyperedges
- Image retrieval: correlations as hyperedges
- Bioinformatics: biochemical interactions as hyperedges
- Hypergraph spectral clustering
- Hypergraph semi-supervised learning

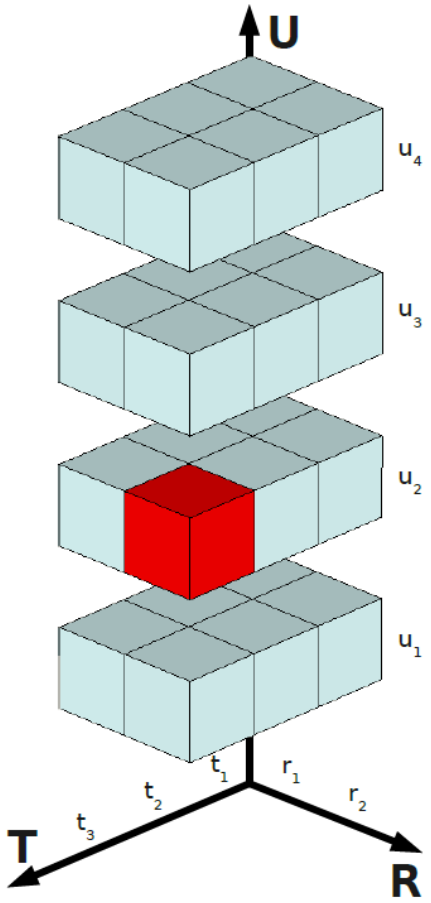
Folksonomies: Hypergraph, Tensor



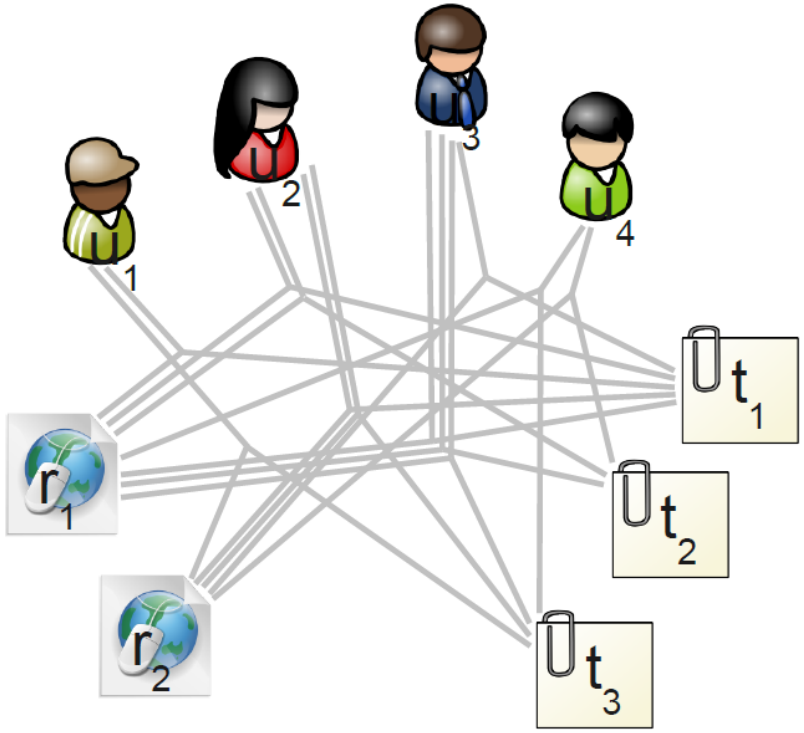


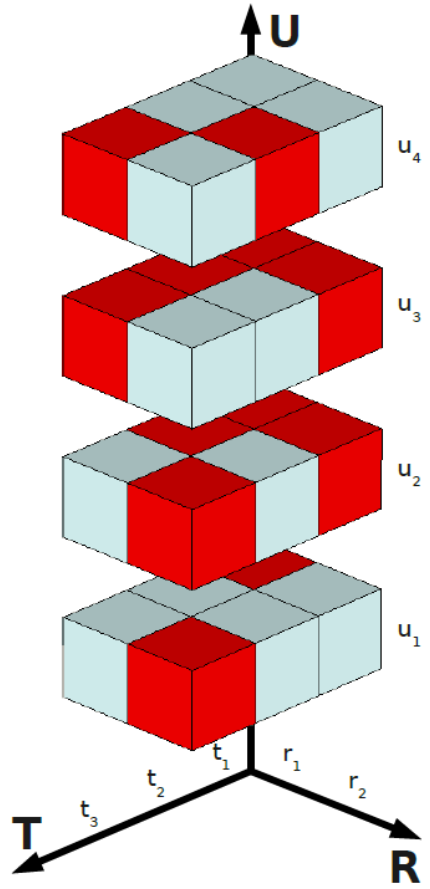
Folksonomies: Hypergraph, Tensor





Folksonomies: Hypergraph, Tensor





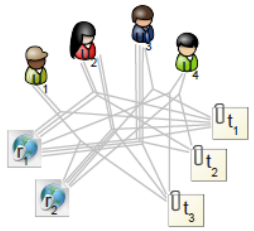
Folksonomies

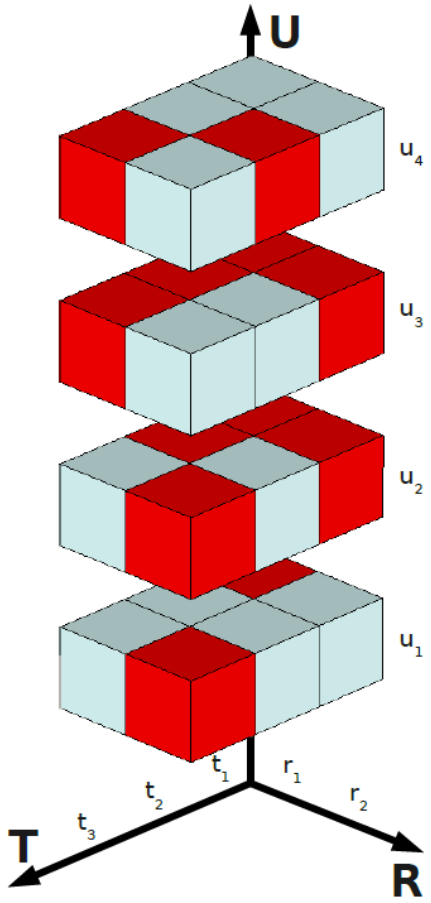
Definition (Folksonomy)

$\mathbb{F} := (U, T, R, Y)$ with

- U, T, R finite sets of users, tags, and resources, resp.
- $Y \subseteq U \times T \times R$ ternary relation

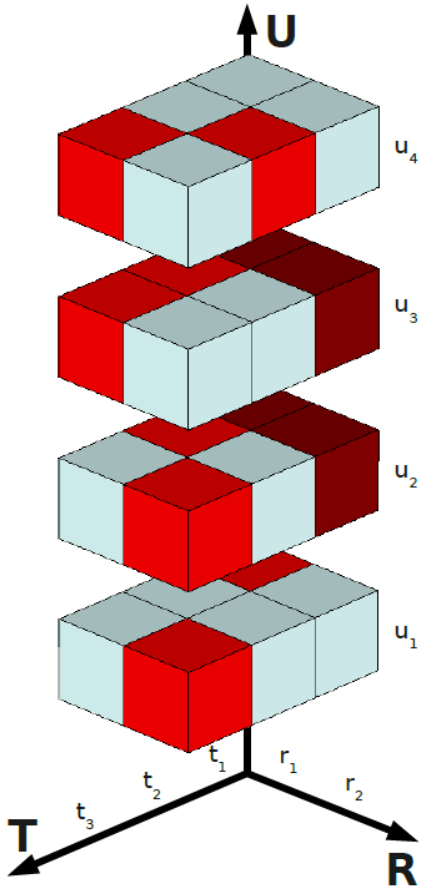
- tripartite hypergraph
- boolean 3-dimensional tensor
- triadic formal context





Folksonomies

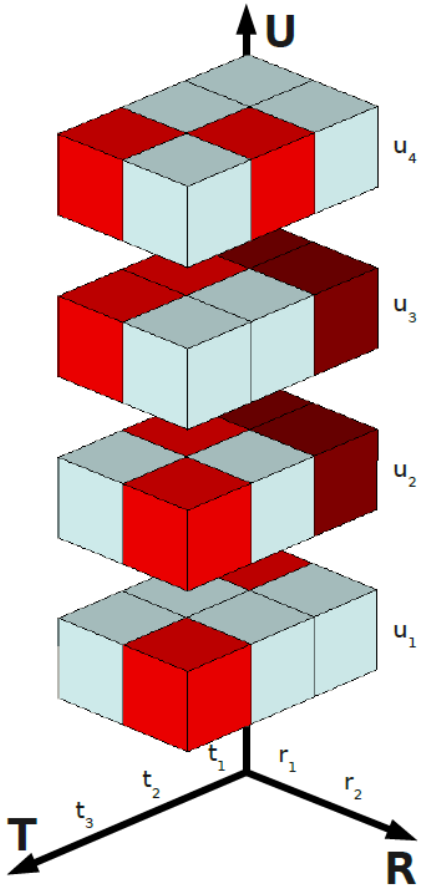
- conceptual clustering of folksonomies
  - find interessting concepts/clusters
  - support browsing, community detection, recommendations
  - get an overview into the structure of a folksonomy

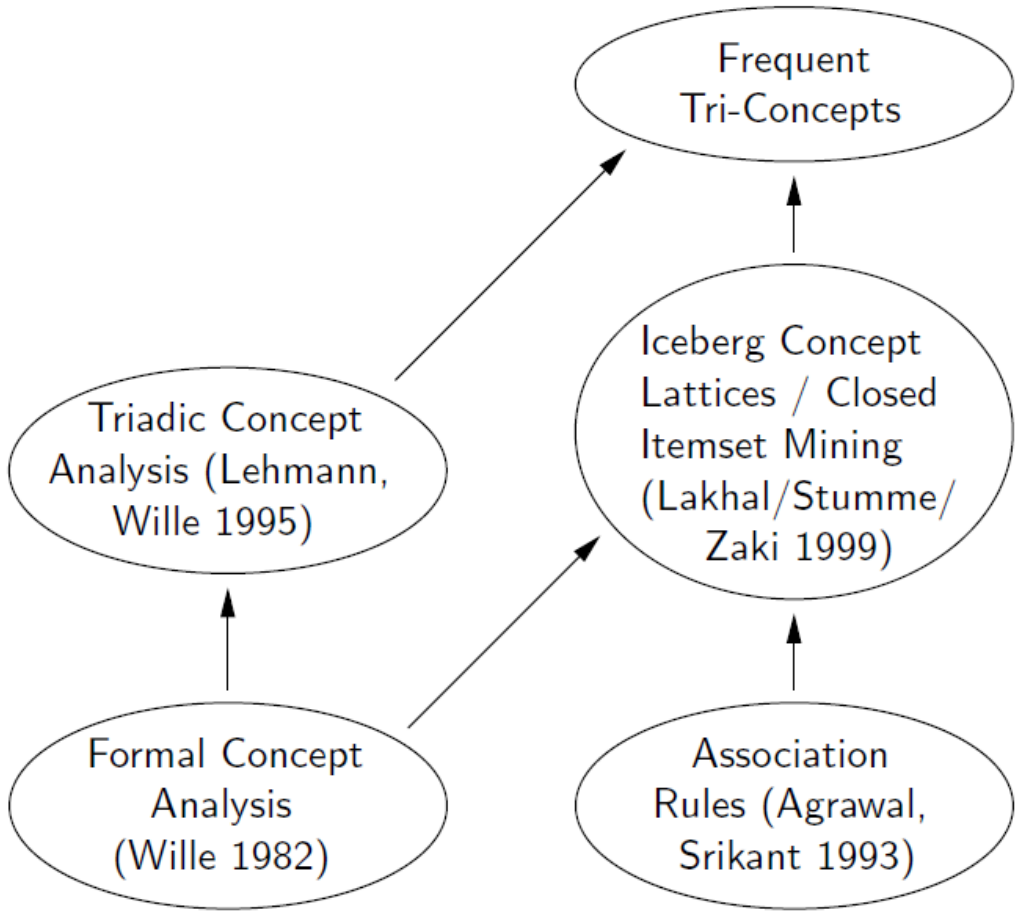


Folksonomies

- conceptual clustering of folksonomies
  - find interessting concepts/clusters
  - support browsing, community detection, recommendations
  - get an overview into the structure of a folksonomy
- *tri-concept* $(A, B, C) \subseteq U \times T \times R$: maximal cuboid in which every user from A has tagged every ressource from C with all tags from B

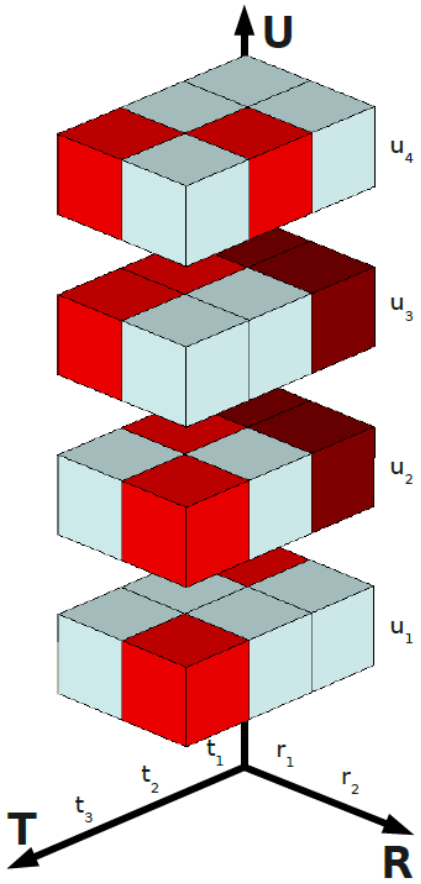
  → shared conceptualization




Frequent
Tri-Concepts
Iceberg Concept
Lattices / Closed
Itemset Mining
(Lakhal/Stumme/
Zaki 1999)
Triadic Concept
Analysis (Lehmann,
Wille 1995)
Formal Concept
Analysis
(Wille 1982)
Association
Rules (Agrawal,
Srikant 1993)

Folksonomies

We regard $\mathbb{F} = (U, T, R, Y)$ as *triadic formal context*.
In general, the elements of U, T and R are then called *objects*, *attributes* and *conditions* and $(u, t, r) \in Y$ is read as "*object* u *has the attribute* t *under condition* r" .



Addendum to the project (1)

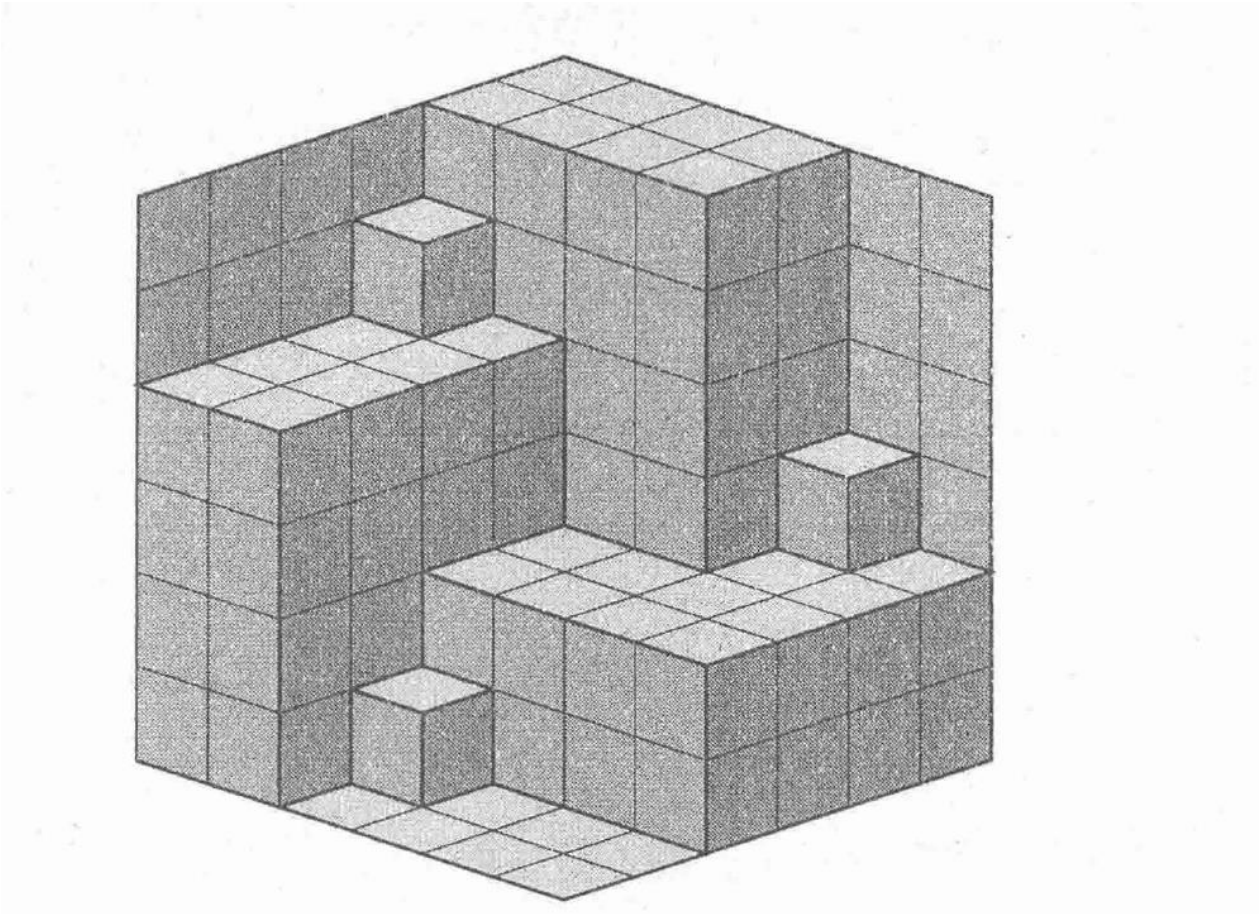
- Find natural examples of triadic data sets and include them in your presentation.
- Discuss their features, structure, etc.

Research question:

Can all data sets be modelled as triadic data sets (for instance by splitting the attribute set?)

Is there any structural difference from data sets having an inherent triadic structure (like folksonomies)?

Tricontext: old example from R. Wille

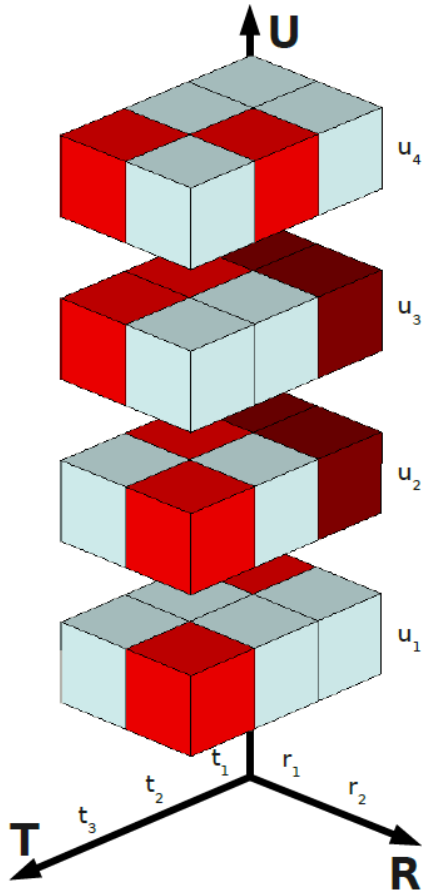


Triadic Formal Concepts

Definition (tri-concept)

triple (A, B, C) with $A \subseteq U$, $B \subseteq T$, $C \subseteq R$ and $A \times B \times C \subseteq Y$, such that none of the three components can be enlarged without violating the condition $A \times B \times C \subseteq Y$.
We call A the *extent*, B the *intent* and C the *modus* of the formal tri-concept.

→ natural extension of formal concepts

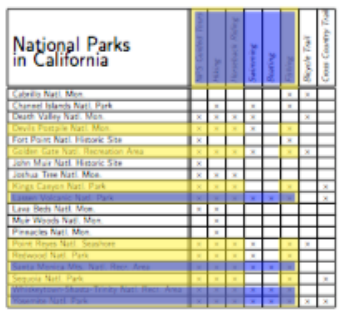


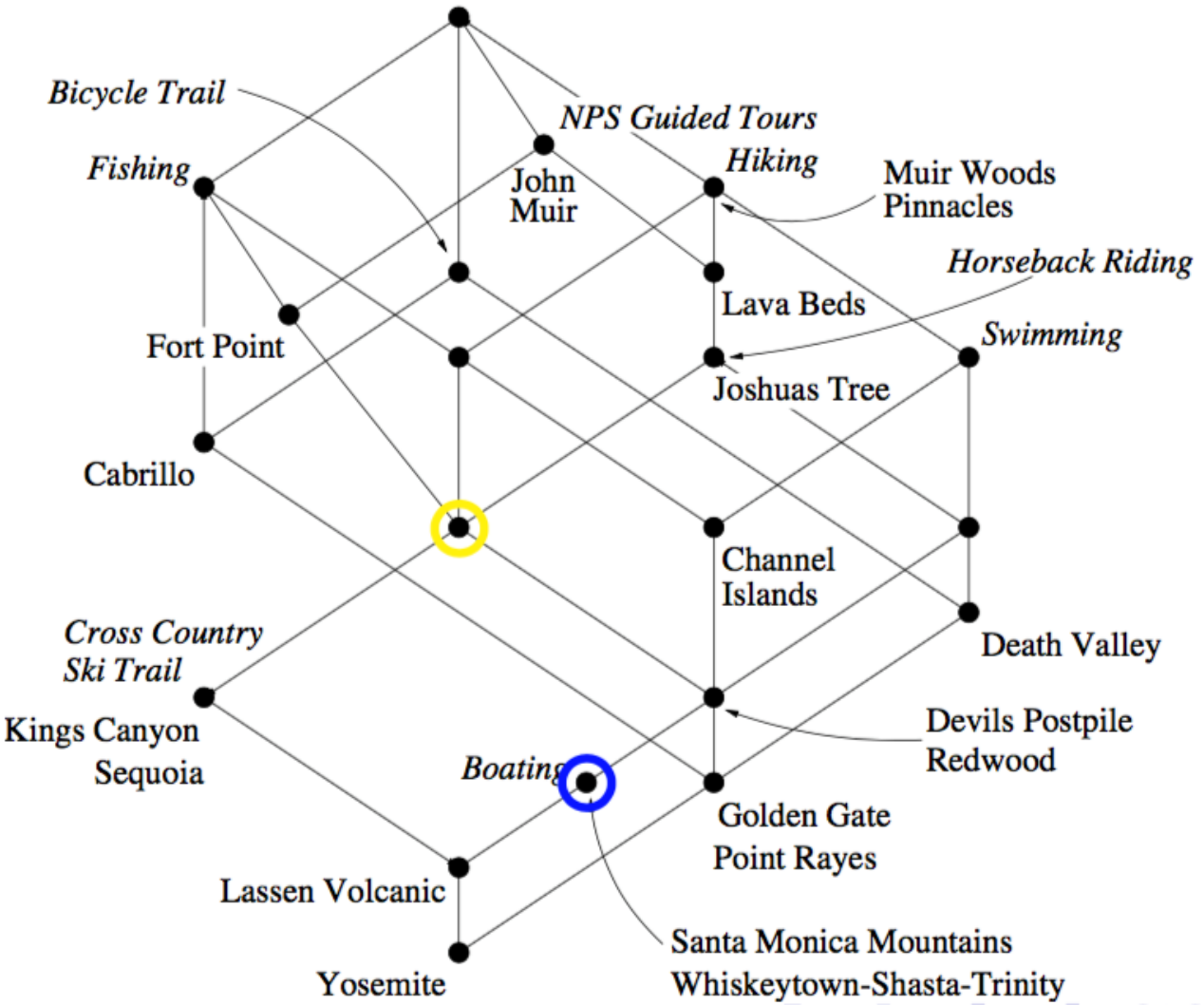
Graphical representation of the triadic conceptual structure

- FCA → concept lattice
- 3FCA → ?
- Recall the construction of the concept lattice using the subconcept-superconcept relationship

Concept Lattice: as Line Diagram

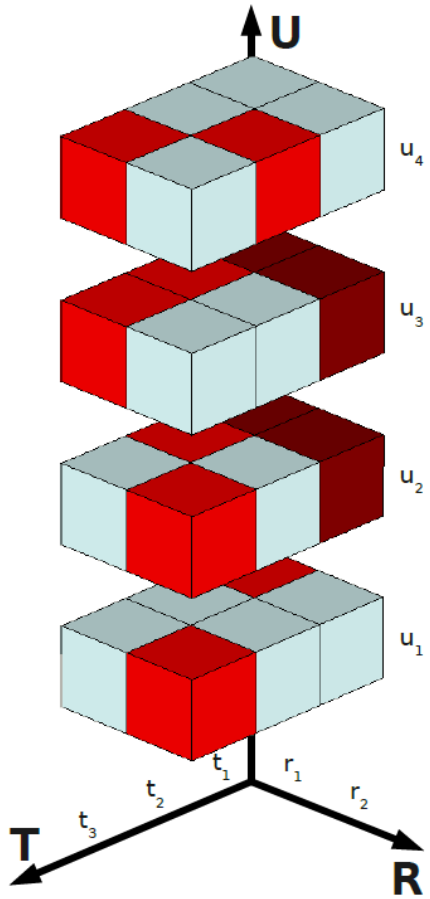
The *concept lattice* for the national park context.





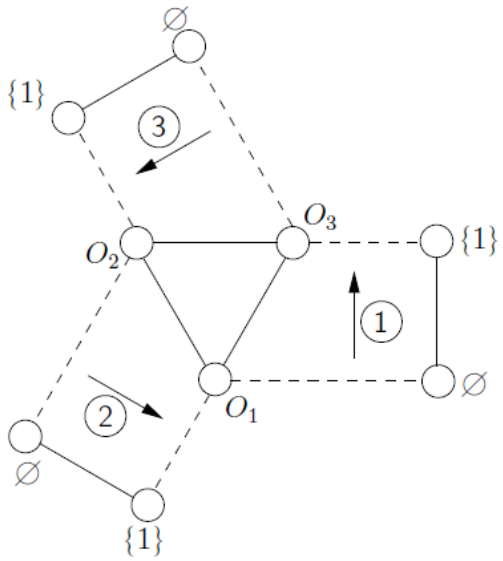
3 FCA → Trilattice

- three quasi orders $\lesssim_1$, $\lesssim_2$, $\lesssim_3$:
  $(A_1, A_2, A_3) \lesssim_i (B_1, B_2, B_3)$
  $:\Leftrightarrow\ A_i \subseteq B_i$, for $i = 1, 2, 3$.
- *not antisymmetric*, i. e. from
  $(A_1, A_2, A_3) \lesssim_i (B_1, B_2, B_3)$ and
  $(B_1, B_2, B_3) \lesssim_i (A_1, A_2, A_3)$ does not
  follow $(A_1, A_2, A_3) = (B_1, B_2, B_3)$
- *concept tri-lattice* $\underline{\mathfrak{B}}(\mathbb{K})$ of the triadic context $\mathbb{K}$
- not a real (mathematical) lattice!



Visualization of Trilattices

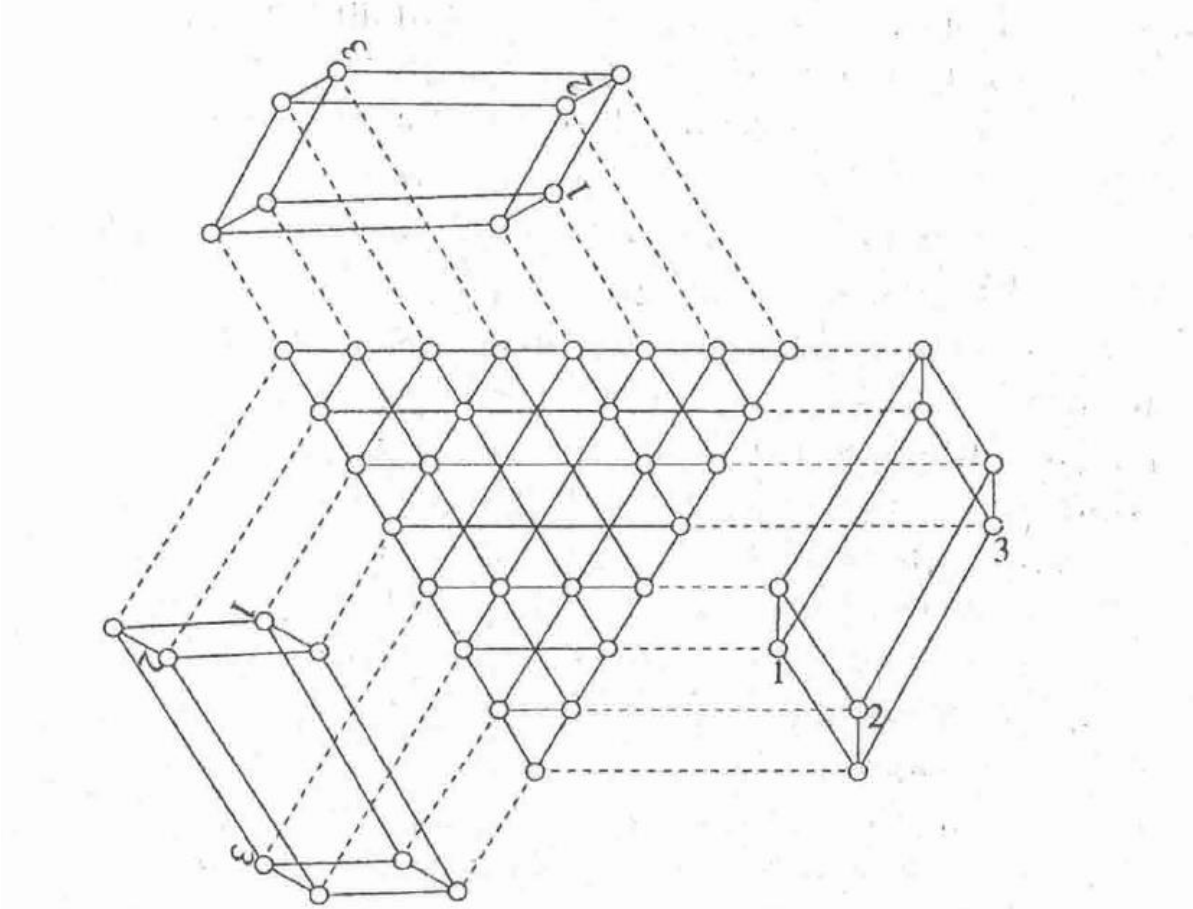
- Since it is not really a lattice, we can not draw a lattice diagram
- Alternative:
  - every quasi-order is written along the edge of a virtual triangle
  - the tri-concepts are drawn into the triangle
- example to the right: smallest non-trivial tri-lattice $\mathbb{B}_3 = \underline{\mathfrak{B}}(\{1\}, \{1\}, \{1\}, \varnothing)$
- visualization not always possible
  - satisfied tetrahedron condition
  - violated Thomson condition



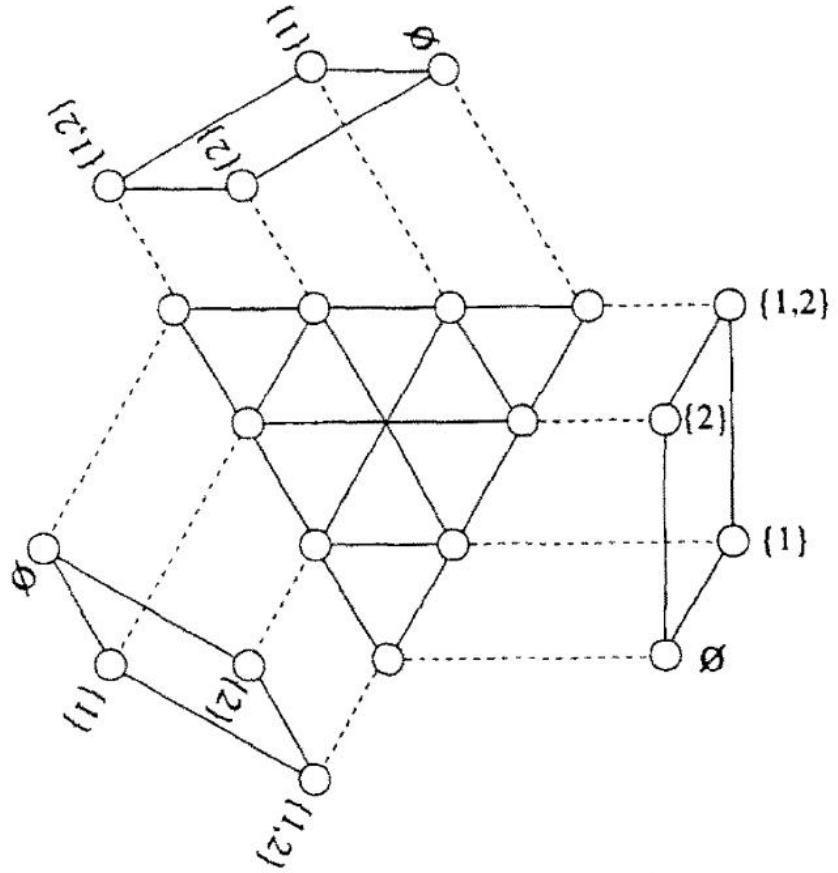
Another example

	1			2			3		
	1	2	3	1	2	3	1	2	3
1		×	×	×	×	×	×	×	×
2	×	×	×	×		×	×	×	×
3	×	×	×	×	×	×	×	×	

Another example



Another example (2)



Another example (3): Trichain TC(5)

	1					2					3					4					5				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	
2	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×		×	×	×		
3	×	×	×	×	×	×	×	×	×	×	×	×	×	×		×	×	×			×	×			
4	×	×	×	×	×	×	×	×	×		×	×	×			×	×				×				
5	×	×	×	×		×	×	×			×	×				×									

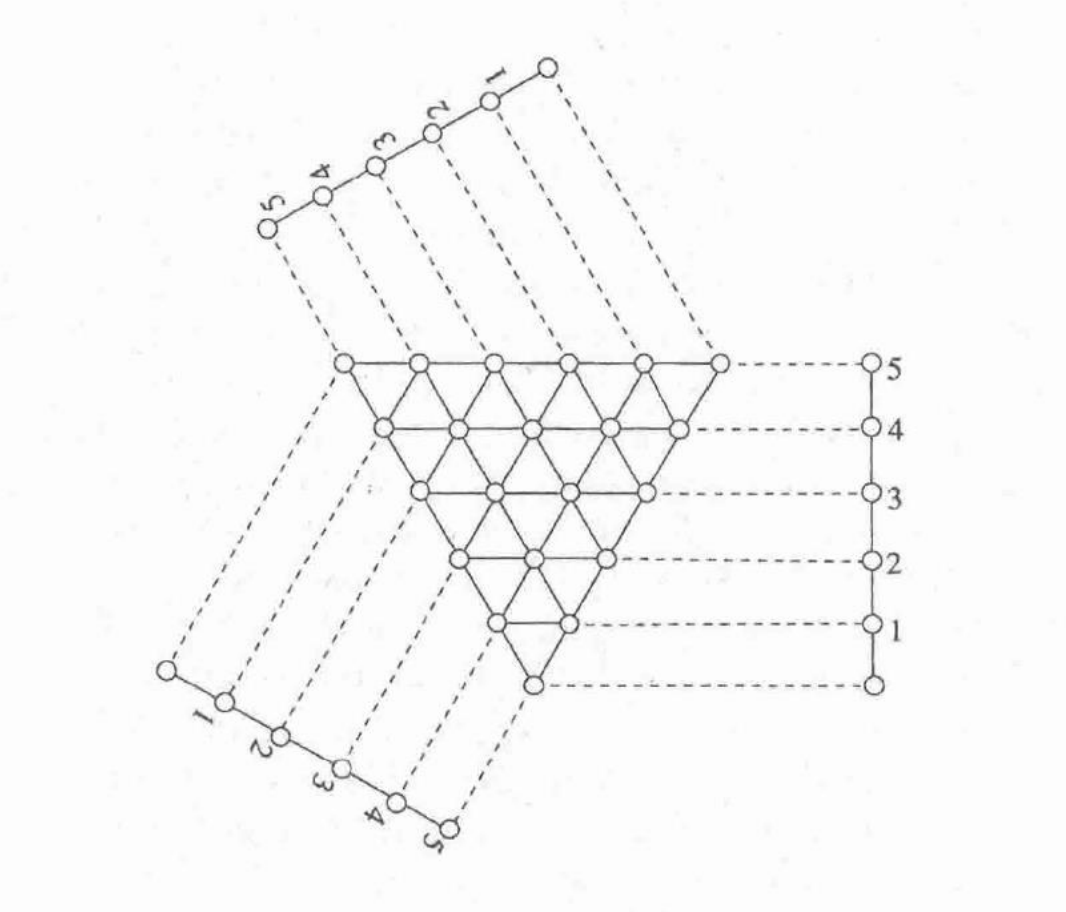
Another example (3): Trichain TC(5)

Exercise:

Give the formula for the incidence relation in TC(5)!

Check the correctness and discover triconcepts in the following trilattice.

Another example (3): Trichain TC(5)



Exercise assignment

	a						b						c						d					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
A	X	X					X	X					X	X				X	X	X		X		
B	X											X	X				X		X				X	
C	X	X	X	X	X	X	X	X	X	X	X	X	X	X				X	X	X				X

A: Superordinate stereotypes
B: Unspecified
C: Subtypes
a: Recall
b: Impression Formation
c: Behavior Prediction
d: Evaluation

1. Physical appearance
2. Political beliefs
3. Attitudes
4. Behavior
5. Traits
6. Situations

Exercise assignment

- Identify triconcepts

Exercise assignment