

# Knowledge Discovery

## Lecture 6: Triadic FCA



# Motivation: Collaborative Tagging Systems

The screenshot shows the Flickr website interface. At the top, the Flickr logo is displayed with the text 'von YAHOO!' and navigation links for 'Startseite', 'Die Tour', 'Registrieren', and 'Entdecken'. A search bar contains the word 'Suchen'. The main heading is 'Entdecken / Tags / nature'. Below this, there are sections for sorting (Neueste, Interessanteste), a cluster of images with the tag 'nature', and a list of related tags such as 'macro', 'flower', 'green', 'landscape', 'trees', 'sky', 'water', 'insect', 'flowers', and 'leaves'. There are also sections for 'Ähnliche Inhalte mit' and 'Sponsoren-Links'.

Sortieren nach:  
Neueste • [Interessanteste](#)

[Cluster mit dem Tag nature](#)  
Entdecken und filtern Sie diese Liste mit dem Tag nature mit unserem tollen Cluster-Feature!

Dazu passende Tags:  
[macro](#), [flower](#), [green](#), [landscape](#), [trees](#), [sky](#), [water](#), [insect](#), [flowers](#), [leaves](#)

Ähnliche Inhalte mit  
[der Yahoo! Bildersuche suchen](#)

Sponsoren-Links  
[PureNature Versand](#)  
Hier finden Sie alles für ein gesundes, allergiefreies Leben!  
[www.PureNature.de](#)  
[Natururlaub in Frankreich](#)  
Natur pur und nachhaltige Konzepte  
[de.francoisguide.com/franccolourism](#)

Image thumbnails and captions:  
- Von [andy.v](#)  
- Von [Tony Reilly1959](#)  
- Von [Tony Reilly1959](#)  
- Von [andy.v](#)  
- Von [Tony Reilly1959](#)  
- Von [ifleish](#)  
- Von [Giorgio](#)  
- Von [Carme M.V.](#)  
- Von [Vanguard1219](#)  
- Von [Vanguard1219](#)  
- Von [Diane &...](#)  
- Von [Claytonia](#)

# Motivation: Collaborative Tagging Systems

The screenshot shows the Vimeo website interface. At the top, there is a navigation bar with the Vimeo logo, a 'Join Vimeo' button, and links for 'Log In', 'Explore', and 'Help'. Below the navigation bar, the main content area is titled 'Videos tagged: nature' with a search input field containing the word 'nature'. Underneath, it indicates 'Videos 1-12 of 12,812'. There are two dropdown menus for filtering: 'Show me newest' and 'videos in thumbnail format'. Below the filters, there are three sponsored links: 'Human Nature Explained' (www.WorldTransformation.com/), 'Visit Lapland' (www.HemavanTarnaby.se), and 'Nurture' (4 hours ago). To the right of the sponsored links is an advertisement for Vimeo. On the left side of the page, there is a sidebar with the Flickr logo and various navigation links. The bottom of the page features two logos: the University of Babeş-Bolyai on the left and the University of Napoca on the right.

# Motivation: Collaborative Tagging Systems

The image displays three side-by-side screenshots of collaborative tagging systems:

- Flickr:** Shows a search page for the tag "nature". It includes a sidebar with navigation links like "Entdecke" and "Cluster mit de", and a main content area with search filters and related content.
- Vimeo:** Shows a page titled "Videos tagged" for the tag "nature". It displays "Videos 1-12 of 12,812" and a "Show me" dropdown menu set to "newest". A video titled "Human Nature Explained" is featured, along with a thumbnail for "Visit Lapland".
- Delicious:** Shows a "Recent nature Bookmarks" page. It features a navigation bar with "Home", "Bookmarks", "People", and "Tags". A search bar is set to "nature". Below, it lists recent bookmarks with titles like "yosemitedigitalphoto's Photos- powered by SmugMug" and "The Point Magazine", each with associated tags and a "SAVE" button.

# Motivation: Collaborative Tagging Systems

http://www.bibsonomy.org/user/jaeschke

**BibSonomy** user :: jaeschke :: tags search (Robert Jäschke) CV EN DE

home myBibSonomy add post groups popular logged in as jaeschke logout

**BOOKMARKS (1141)**

- Twitter Calendar**  
http://statuscalendar.cs.washington.edu/  
17 hours and 29 minutes ago by jaeschke  
information named entity ner calendar twitter e...  
★★★★★ (0)
- Feature of the week: CSL via REST-API**  
http://blog.bibsonomy.org/2013/01/feature-of-week-csl-via-rest-a...  
3 days and an hour ago by jaeschke  
bibsonomynews ftw csl bibsonomy rest  
★★★★★ (0)
- prisma.de: It Might Get Loud**  
http://www.prisma.de/film/2008\_t\_might\_get\_loud/fernsehen.html  
4 days ago by jaeschke as private  
program tv music  
★★★★★ (0)
- ORCID: Robert Jäschke**  
http://orcid.org/0000-0003-3271-9653  
7 days ago by jaeschke  
orcid science myown research publication  
★★★★★ (0)
- BibSonomy**  
http://www.l3s.uni-hannover.de/~jaeschke/bibsonomy/  
7 days ago by jaeschke  
tagging collaborative social bookmarking bibso...

**PUBLICATIONS (726)**

- The Wiki way: quick collaboration on the Web**  
Bo Leuf, and Ward Cunningham. Addison-Wesley, London, (Marc...  
7 days ago by jaeschke  
collaboration wiki management web knowledge  
★★★★★ (0)
- Best Practices for Scientific Computing**  
Greg Wilson, D. A. Aruloh, C. Titus Brown, Neil P. Chue Hong, Me...  
9 days ago by jaeschke  
programming science computing research  
★★★★★ (0)
- Understanding the internet: a socio-cultural ..**  
Bridgette Vessels. Palgrave Macmillan, Houndmills, Basingstok...  
9 days ago by jaeschke  
stair understanding culture internet  
★★★★★ (0)
- The no-nonsense guide to equality**  
Daniel Döring, Kate Pickett, and Richard G. Wilkinson. New Interna...  
9 days ago by jaeschke  
stair income justice guide equality  
★★★★★ (0)
- So You Think You Know About Britain?**  
Daniel Döring. Constable & Robinson, London, (2011)  
9 days ago by jaeschke  
stair britain geography

**concepts**  
(show all | hide all)

- author ← newman
- conference ← ecai ecmpldd gvd iccs icdm icfea  
recsys
- folksonomy ← bookmarking tagging
- geo ← gps map utm
- howto ← manual reference tutorial
- location ← smat berlin bitterfeld bied cele  
dagstuhl dresden europe frankfurt hannover  
hessen italien kassel london magdeburg ort  
saarland sachsen sachsen\_anhalt toulouse  
tübingen wadern wittenberg würzburg
- ort ← location
- programming ← ada c fortran java perl prolog  
python ruby
- protocol ← ftp http smtp
- researcher ← devadze shannon turing
- science ← chemistry math
- software ← apache beagle cocoa debian eclipse  
firefox haystack nextstep photoshop protege  
thunderbird wala wine word x11 zope

- manage your web bookmarks and publication references
- open for the public since beginning of 2006, > 5 000 active users
- developed and operated at L3S Research Center





# Folksonomies: Hypergraph

- What is a hypergraph?



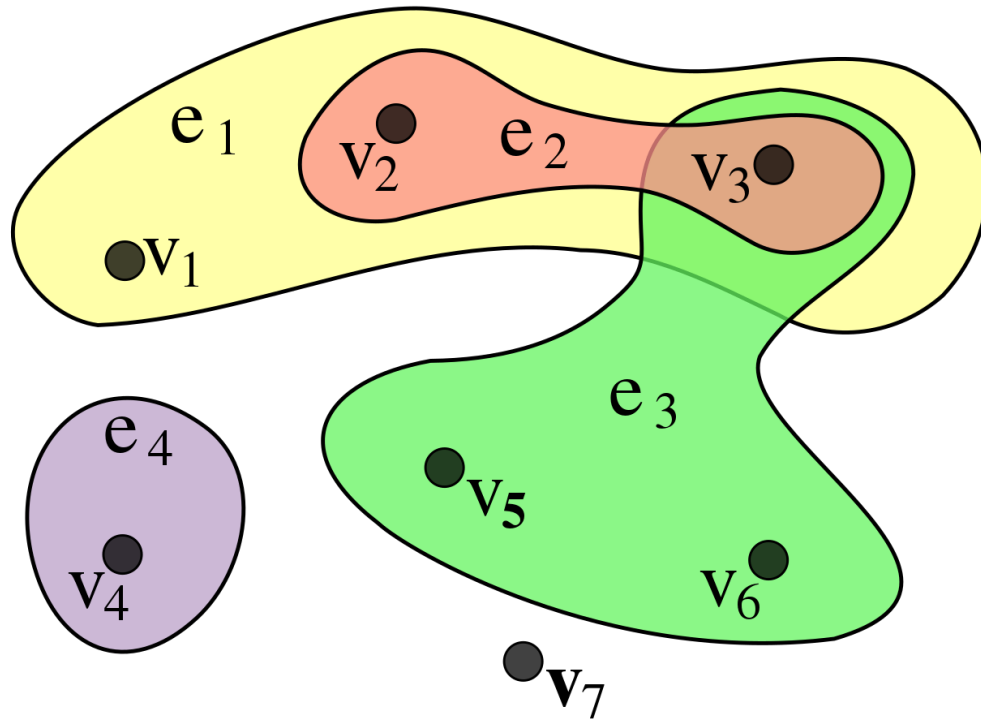
# Hypergraph

- Generalization of a graph in which an **edge** can **join any number of vertices**
- $H=(X,E)$ .
- $X$  = set of vertices;
- $E$  subset of  $P(X)$  (but not empty)





# Hypergraph



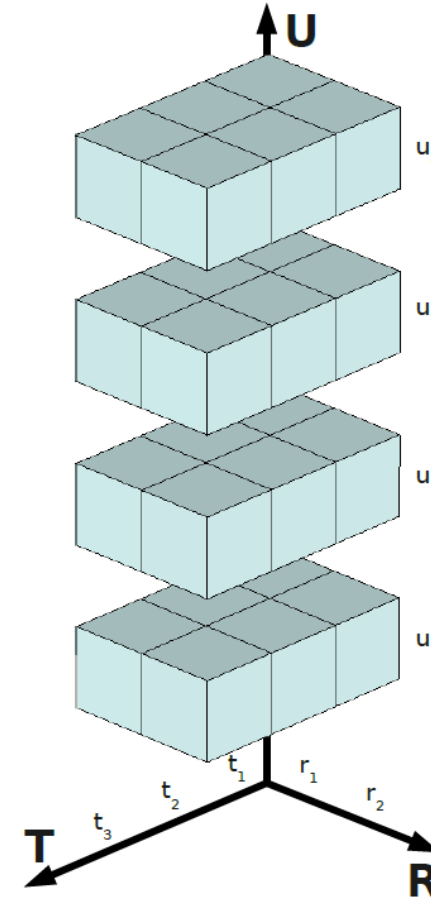
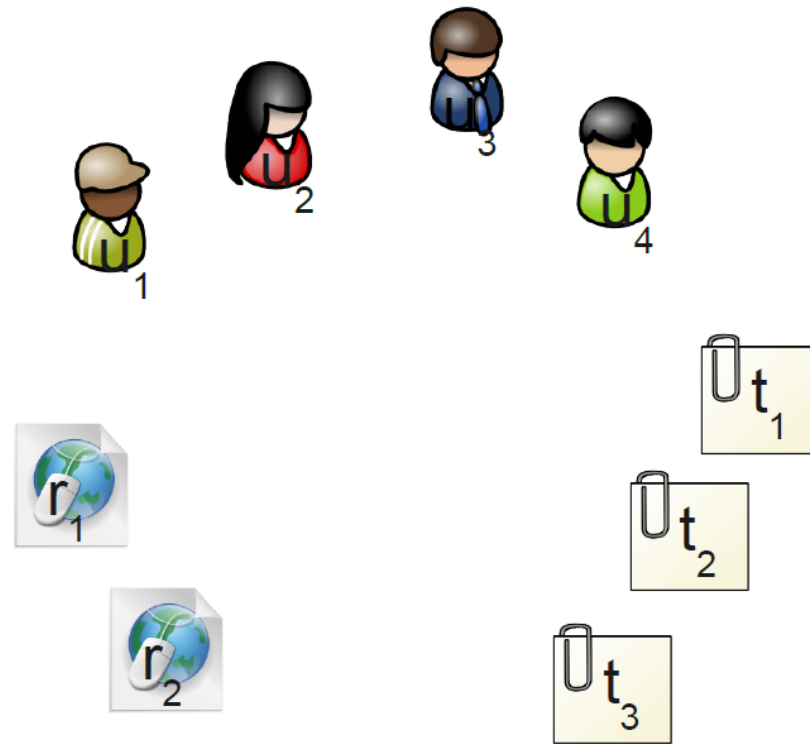
An example of a hypergraph, with  $X = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$  and  $E = \{e_1, e_2, e_3, e_4\} = \{\{v_1, v_2, v_3\}, \{v_2, v_3\}, \{v_3, v_5, v_6\}, \{v_4\}\}$ . Here, edges do not just connect two vertices but several, and are represented by colors.

# Hypergraph applications

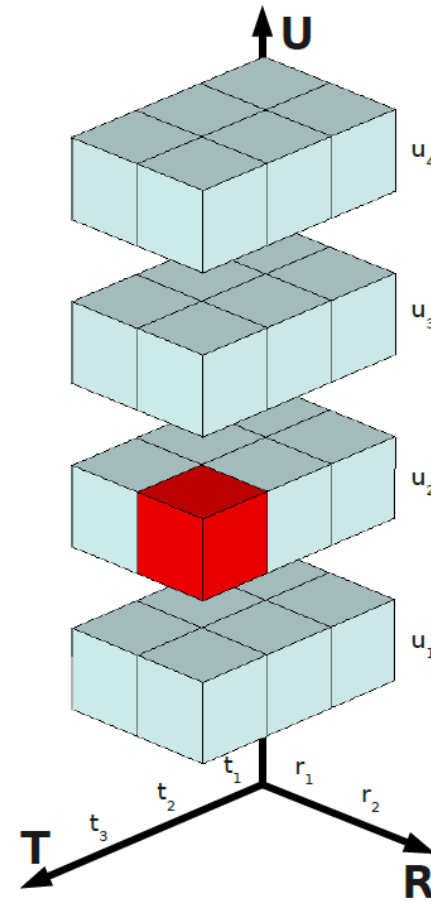
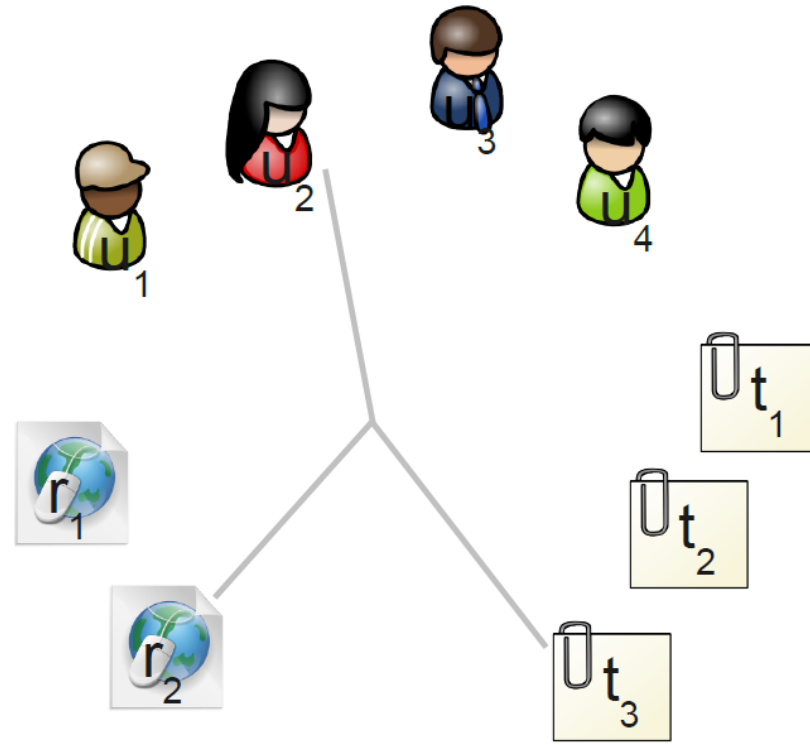
- Machine learning
- Recommender systems: communities as hyperedges
- Image retrieval: correlations as hyperedges
- Bioinformatics: biochemical interactions as hyperedges
- Hypergraph spectral clustering
- Hypergraph semi-supervised learning



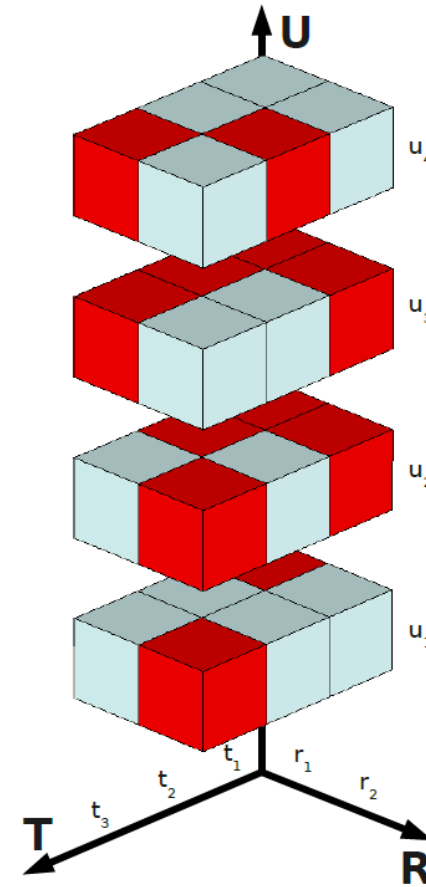
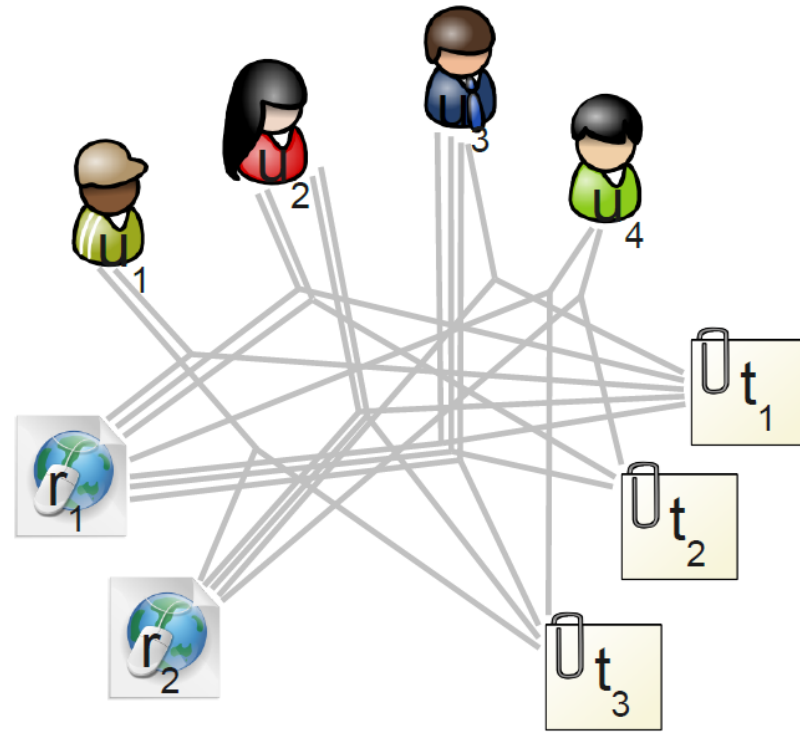
# Folksonomies: Hypergraph, Tensor



# Folksonomies: Hypergraph, Tensor



# Folksonomies: Hypergraph, Tensor



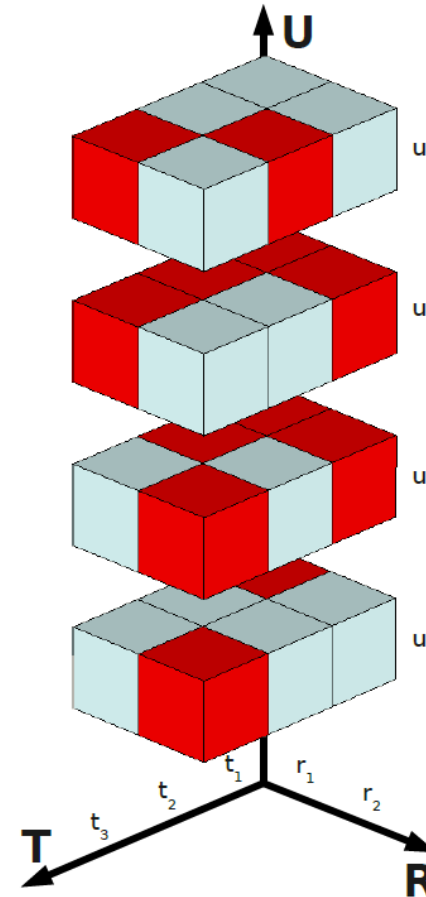
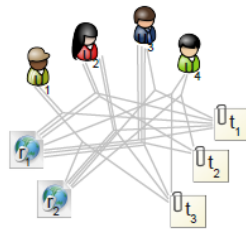
# Folksonomies

## Definition (Folksonomy)

$\mathbb{F} := (U, T, R, Y)$  with

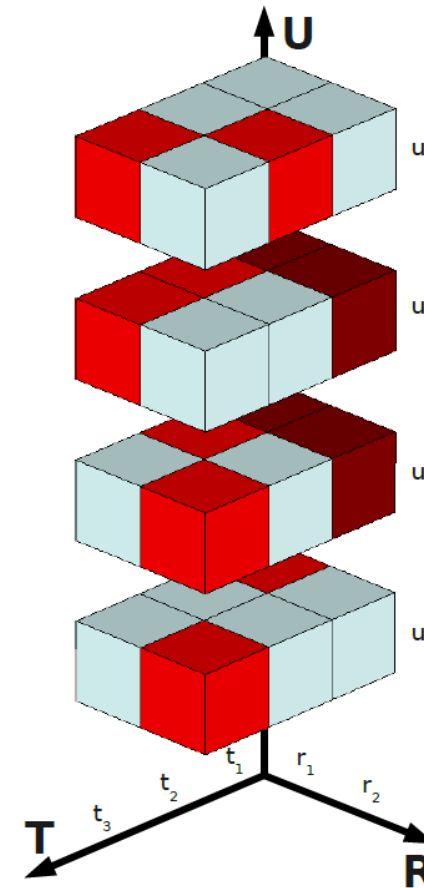
- $U, T, R$  finite sets of users, tags, and resources, resp.
- $Y \subseteq U \times T \times R$  ternary relation

- tripartite hypergraph
- boolean 3-dimensional tensor
- triadic formal context



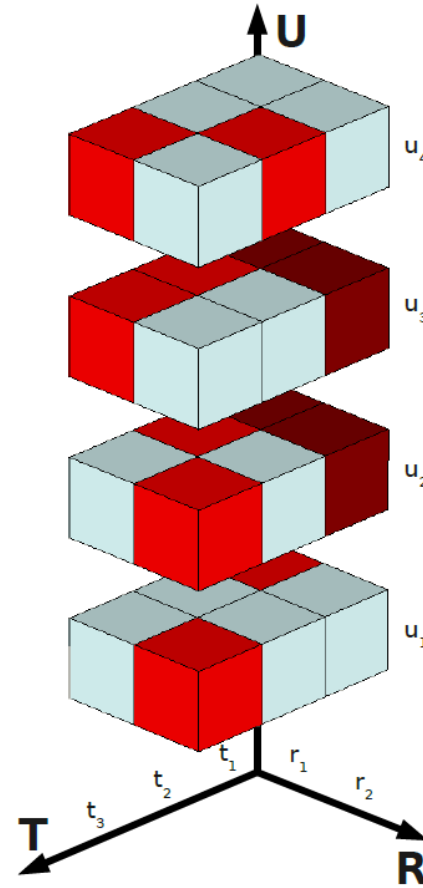
# Folksonomies

- conceptual clustering of folksonomies
  - find interesting concepts/clusters
  - support browsing, community detection, recommendations
  - get an overview into the structure of a folksonomy

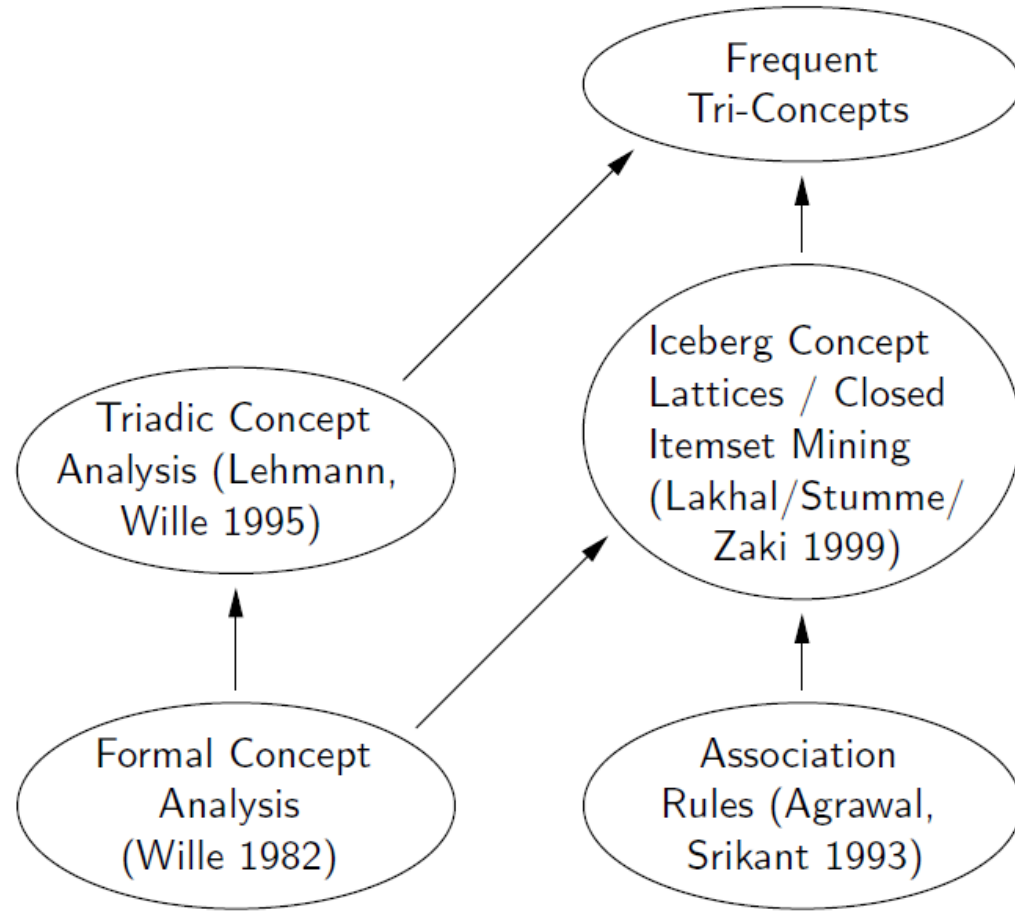


# Folksonomies

- conceptual clustering of folksonomies
  - find interesting concepts/clusters
  - support browsing, community detection, recommendations
  - get an overview into the structure of a folksonomy
- *tri-concept*  $(A, B, C) \subseteq U \times T \times R$ : maximal cuboid in which every user from  $A$  has tagged every resource from  $C$  with all tags from  $B$ 
  - shared conceptualization



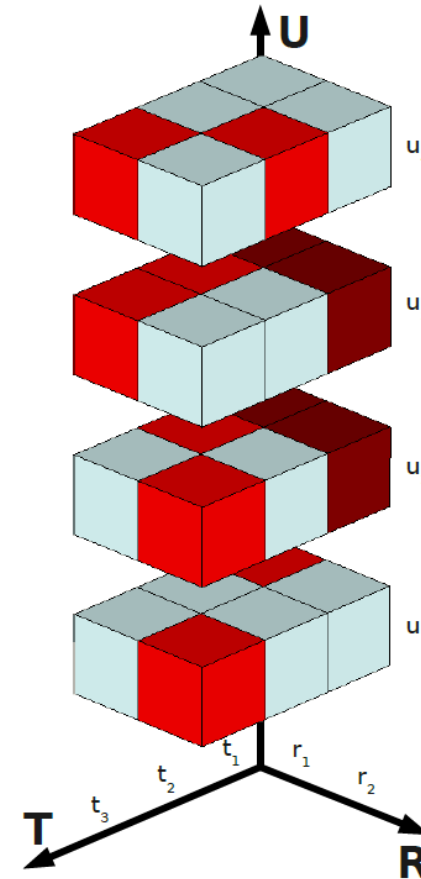




# Folksonomies

We regard  $\mathbb{F} = (U, T, R, Y)$  as *triadic formal context*.

In general, the elements of  $U$ ,  $T$  and  $R$  are then called *objects*, *attributes* and *conditions* and  $(u, t, r) \in Y$  is read as “object  $u$  has the attribute  $t$  under condition  $r$ ”.



# Addendum to the project (1)

- Find natural examples of triadic data sets and include them in your presentation.
- Discuss their features, structure, etc.

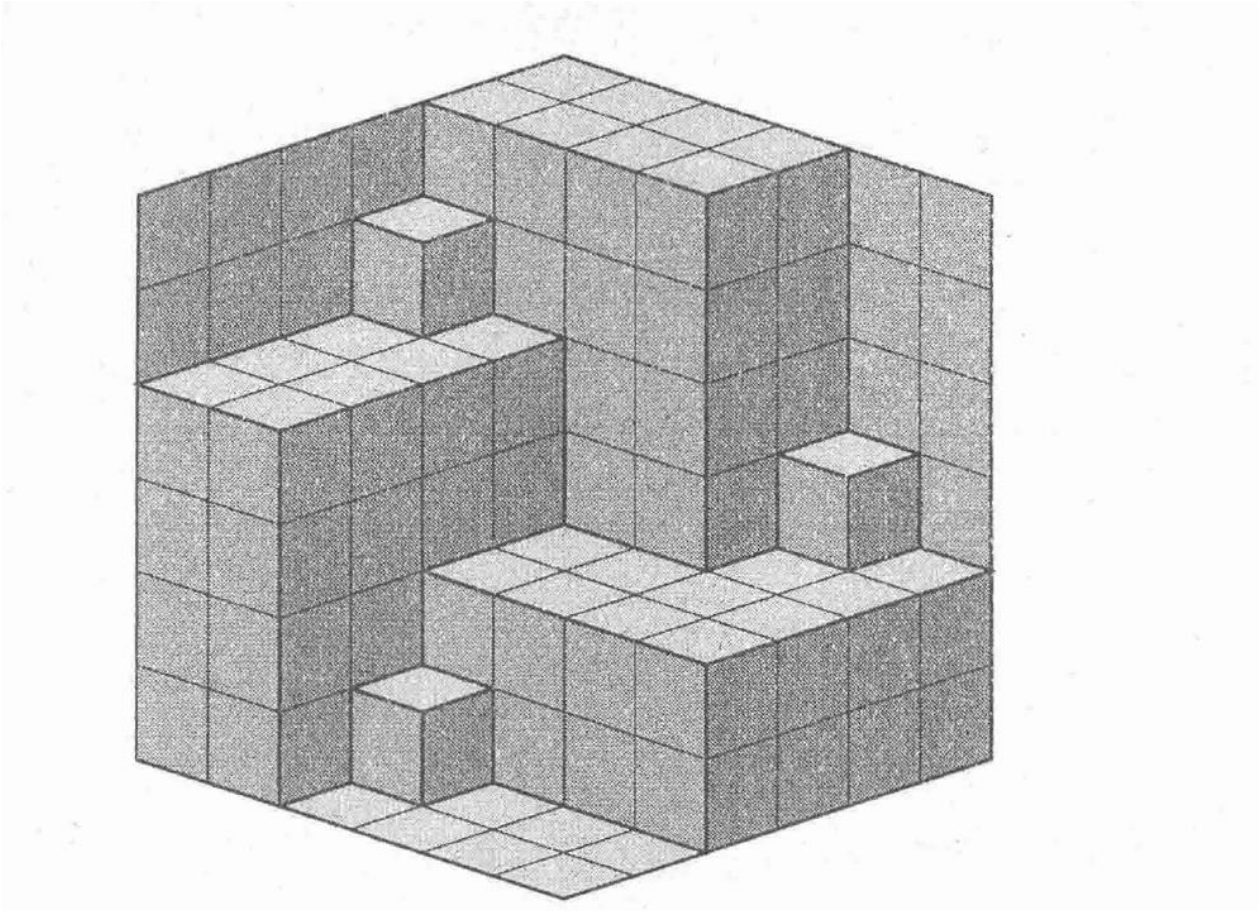
## Research question:

Can all data sets be modelled as triadic data sets (for instance by splitting the attribute set?)

Is there any structural difference from data sets having an inherent triadic structure (like folksonomies)?



# Tricontext: old example from R. Wille



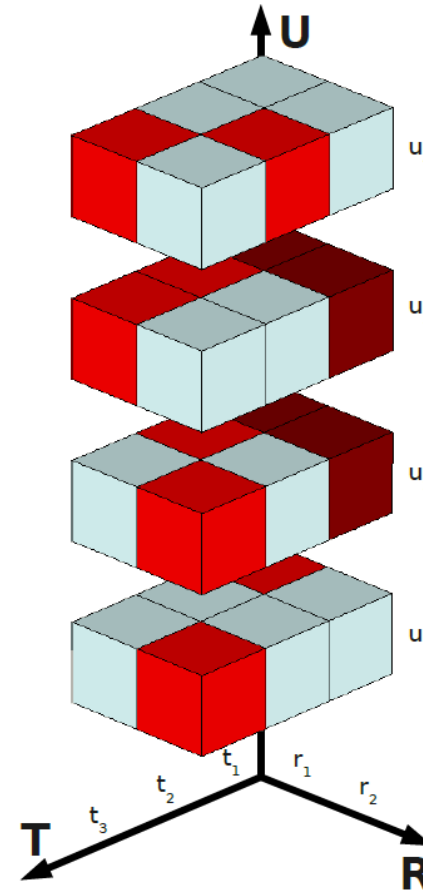
# Triadic Formal Concepts

## Definition (tri-concept)

triple  $(A, B, C)$  with  $A \subseteq U$ ,  $B \subseteq T$ ,  $C \subseteq R$  and  $A \times B \times C \subseteq Y$ , such that none of the three components can be enlarged without violating the condition  $A \times B \times C \subseteq Y$ .

We call  $A$  the *extent*,  $B$  the *intent* and  $C$  the *modus* of the formal tri-concept.

→ natural extension of formal concepts



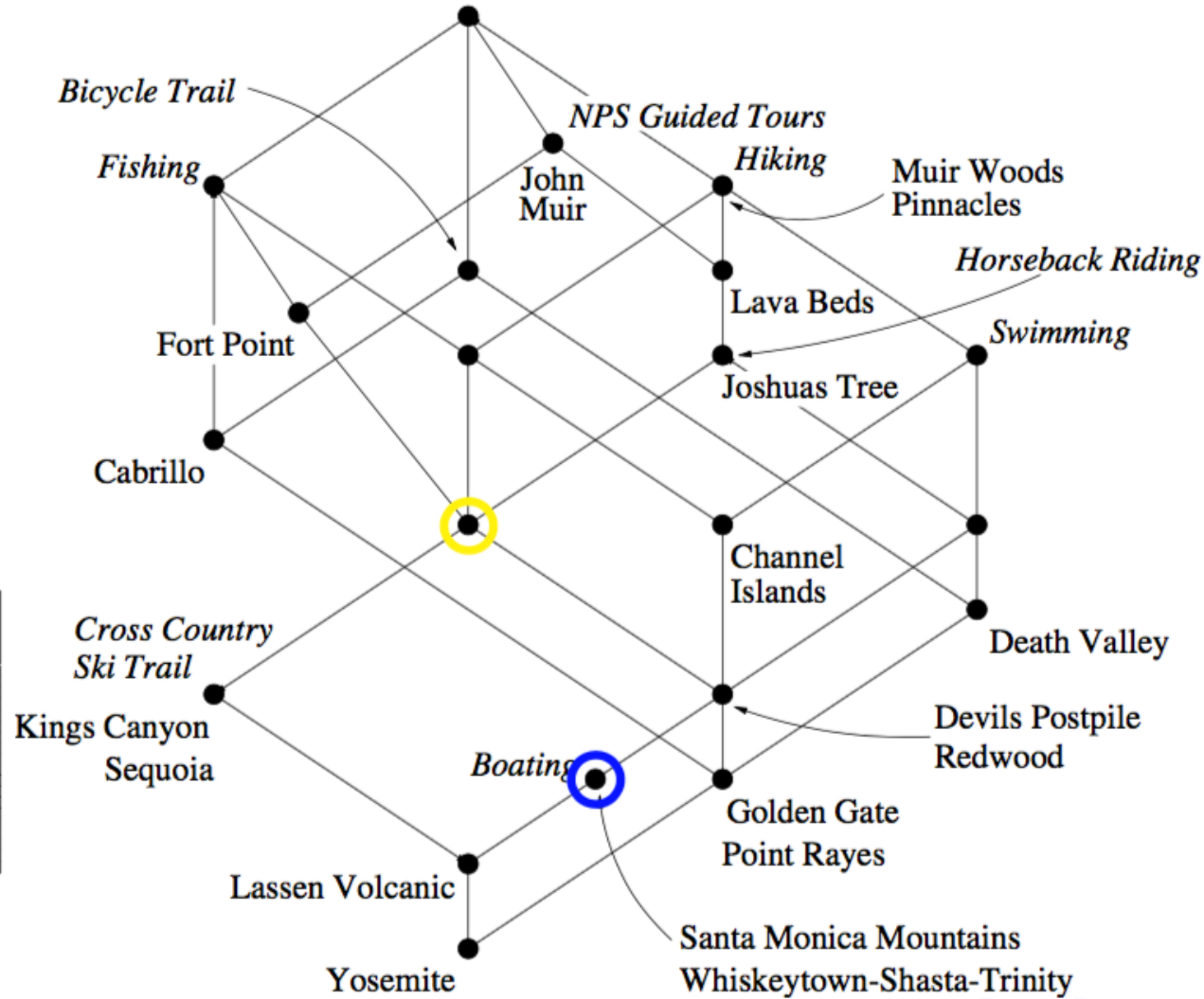
# Graphical representation of the triadic conceptual structure

- FCA  $\rightarrow$  concept lattice
- 3FCA  $\rightarrow$  ?
- Recall the construction of the concept lattice using the subconcept-superconcept relationship



# Concept Lattice: as Line Diagram

The *concept lattice* for the national park context.

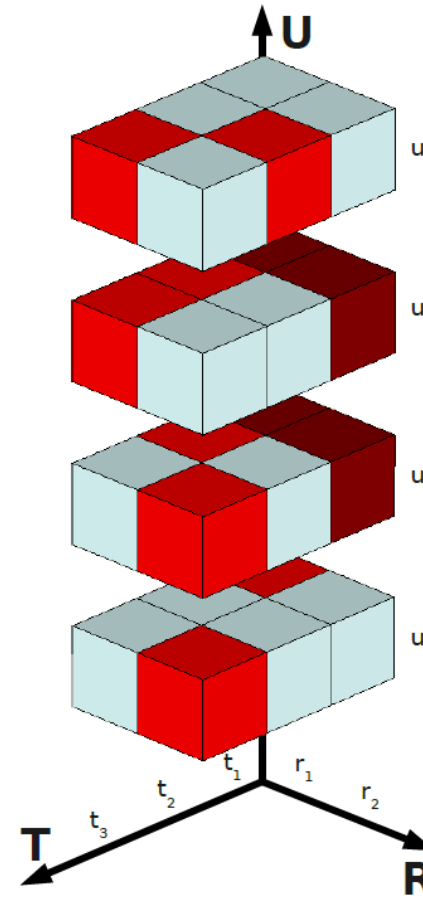


National Parks in California	Bicycle Trail	Fishing	Hiking	Swimming	Boating	Cross Country Ski Trail
Cabrillo Natl. Mon.	x	x	x	x	x	x
Channel Islands Natl. Park	x	x	x	x	x	x
Death Valley Natl. Mon.	x	x	x	x	x	x
Devils Postpile Natl. Mon.	x	x	x	x	x	x
Fort Point Natl. Historic Site	x	x	x	x	x	x
Golden Gate Natl. Recreation Area	x	x	x	x	x	x
John Muir Natl. Historic Site	x	x	x	x	x	x
Joshua Tree Natl. Mon.	x	x	x	x	x	x
Kings Canyon Natl. Park	x	x	x	x	x	x
Lava Beds Natl. Mon.	x	x	x	x	x	x
Muir Woods Natl. Mon.	x	x	x	x	x	x
Pinnacles Natl. Mon.	x	x	x	x	x	x
Santa Monica Mountains Natl. Mon.	x	x	x	x	x	x
Shasta-Trinity Natl. Mon.	x	x	x	x	x	x
Whiskeytown Natl. Mon.	x	x	x	x	x	x
Yosemite Natl. Park	x	x	x	x	x	x
Yosemite National Park	x	x	x	x	x	x



# 3 FCA $\rightarrow$ Trilattice

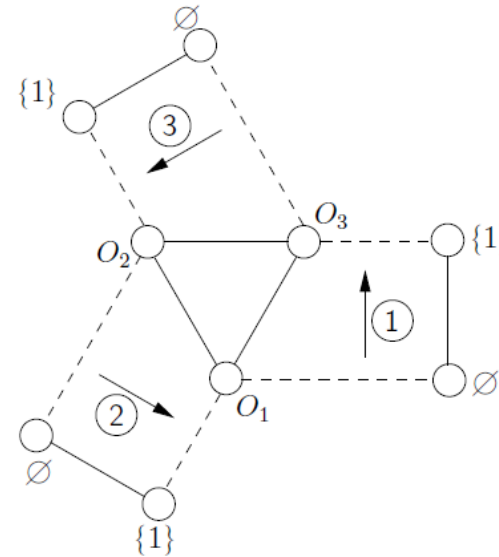
- three quasi orders  $\lesssim_1, \lesssim_2, \lesssim_3$ :  
 $(A_1, A_2, A_3) \lesssim_i (B_1, B_2, B_3)$   
 $:\Leftrightarrow A_i \subseteq B_i$ , for  $i = 1, 2, 3$ .
- *not antisymmetric*, i. e. from  
 $(A_1, A_2, A_3) \lesssim_i (B_1, B_2, B_3)$  and  
 $(B_1, B_2, B_3) \lesssim_i (A_1, A_2, A_3)$  does not  
follow  $(A_1, A_2, A_3) = (B_1, B_2, B_3)$
- *concept tri-lattice*  $\underline{\mathfrak{B}}(\mathbb{K})$  of the triadic  
context  $\mathbb{K}$
- not a real (mathematical) lattice!





# Visualization of Trilattices

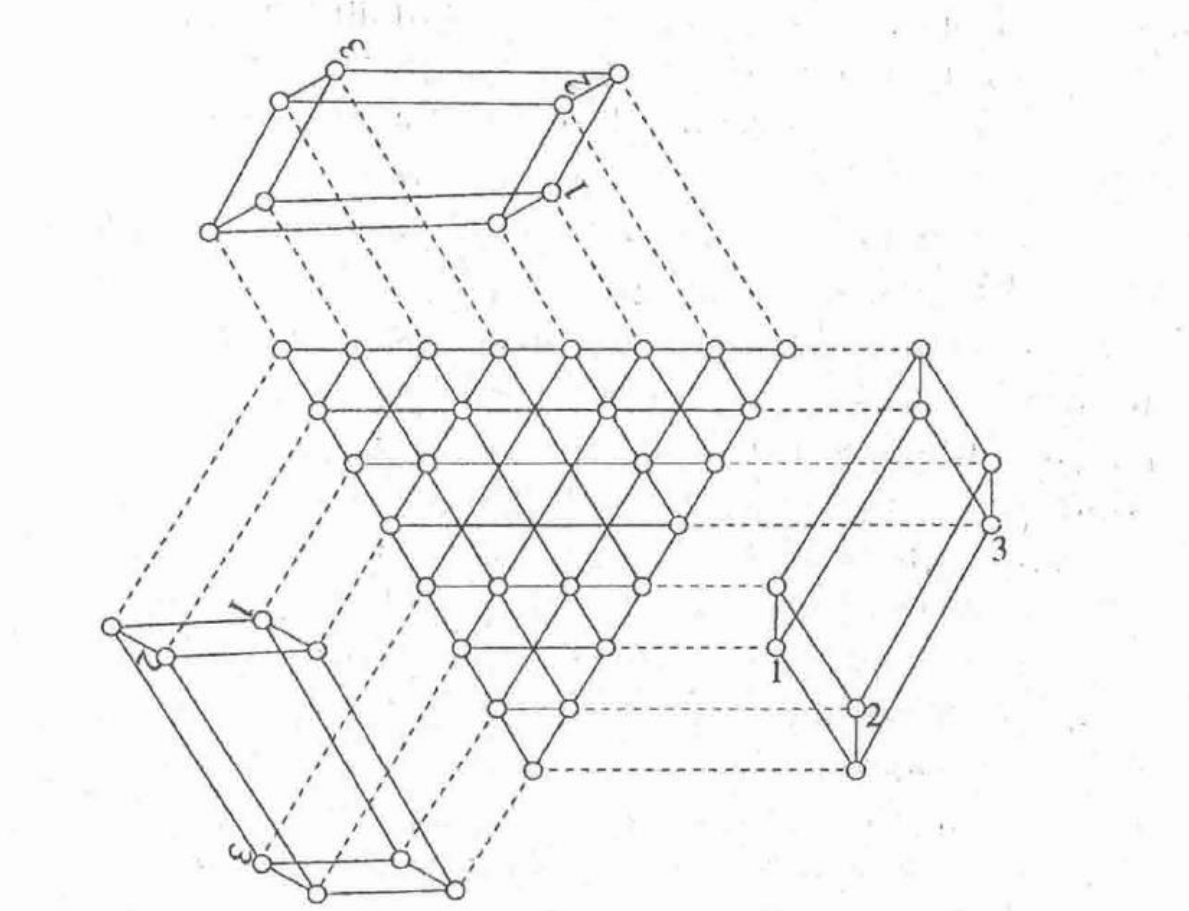
- Since it is not really a lattice, we can not draw a lattice diagram
- Alternative:
  - every quasi-order is written along the edge of a virtual triangle
  - the tri-concepts are drawn into the triangle
- example to the right: smallest non-trivial tri-lattice  
 $\mathbb{B}_3 = \underline{\mathfrak{B}}(\{1\}, \{1\}, \{1\}, \emptyset)$
- visualization not always possible
  - satisfied tetrahedron condition
  - violated Thomson condition



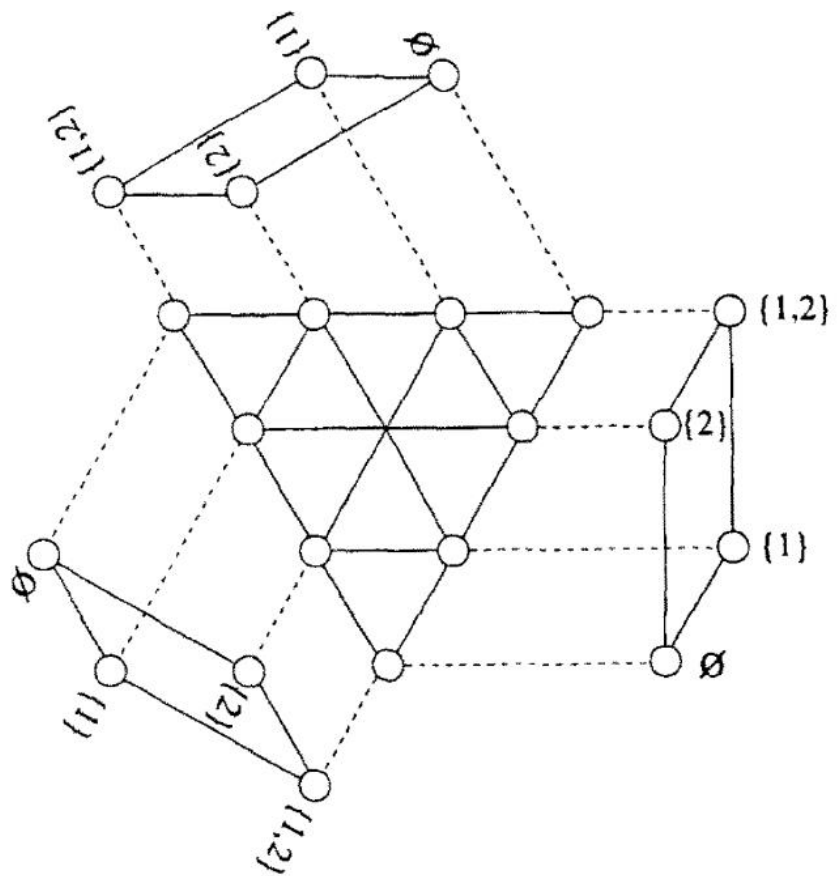
# Another example

	1			2			3		
	1	2	3	1	2	3	1	2	3
1		x	x	x	x	x	x	x	x
2	x	x	x	x			x	x	x
3	x	x	x	x	x	x	x	x	

# Another example



# Another example (2)



# Another example (3): Trichain TC(5)

	1					2					3					4					5				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	
2	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x		
3	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x			x	x			
4	x	x	x	x	x	x	x	x	x		x	x	x			x	x				x				
5	x	x	x	x		x	x	x			x	x				x									



# Another example (3): Trichain TC(5)

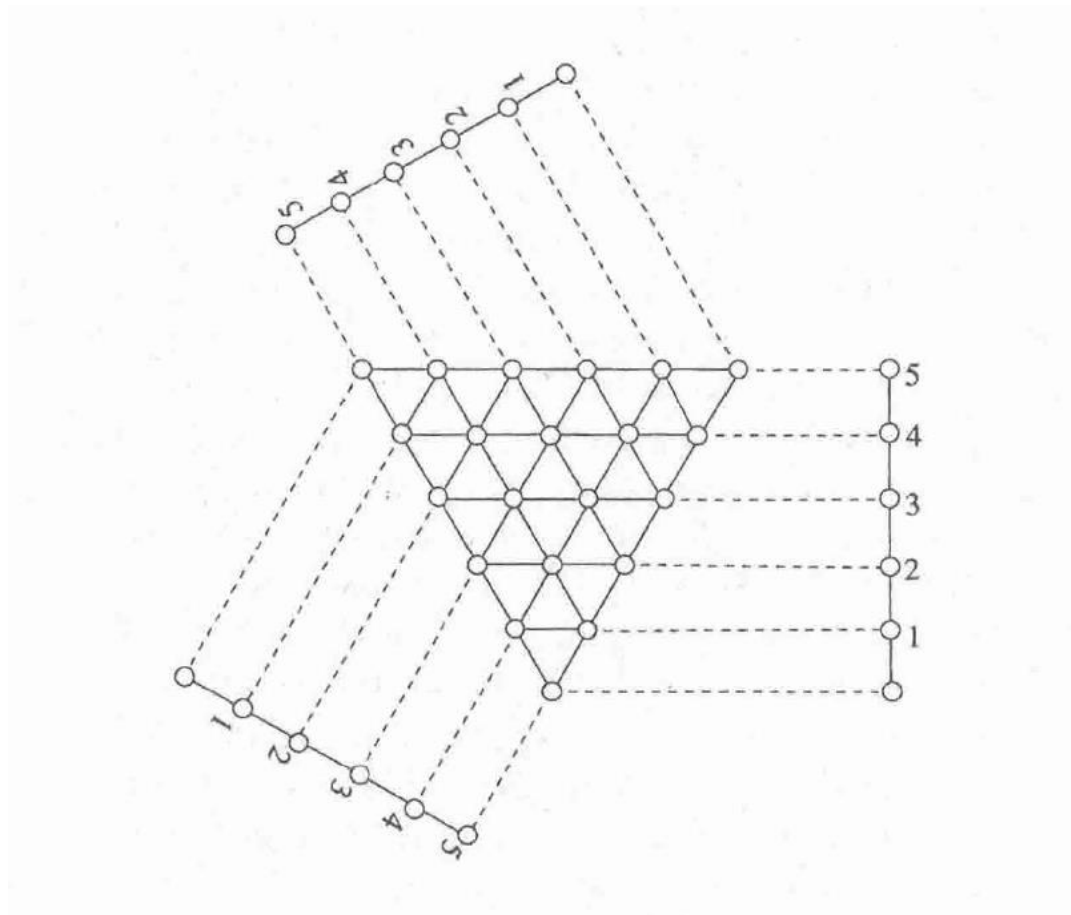
## Exercise:

Give the formula for the incidence relation in TC(5)!

Check the correctness and discover triconcepts in the following trilattice.



# Another example (3): Trichain TC(5)



# Exercise assignment

	a						b						c						d											
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6						
A	X	X					X	X					X	X					X	X	X		X							
B	X											X	X					X		X					X					
C	X	X	X	X	X	X	X	X	X	X	X	X	X	X					X	X	X									X

A: Superordinate stereotypes

B: Unspecified

C: Subtypes

a: Recall

b: Impression Formation

c: Behavior Prediction

d: Evaluation

1. Physical appearance

2. Political beliefs

3. Attitudes

4. Behavior

5. Traits

6. Situations



# Exercise assignment

- Identify triconcepts



# Exercise assignment

