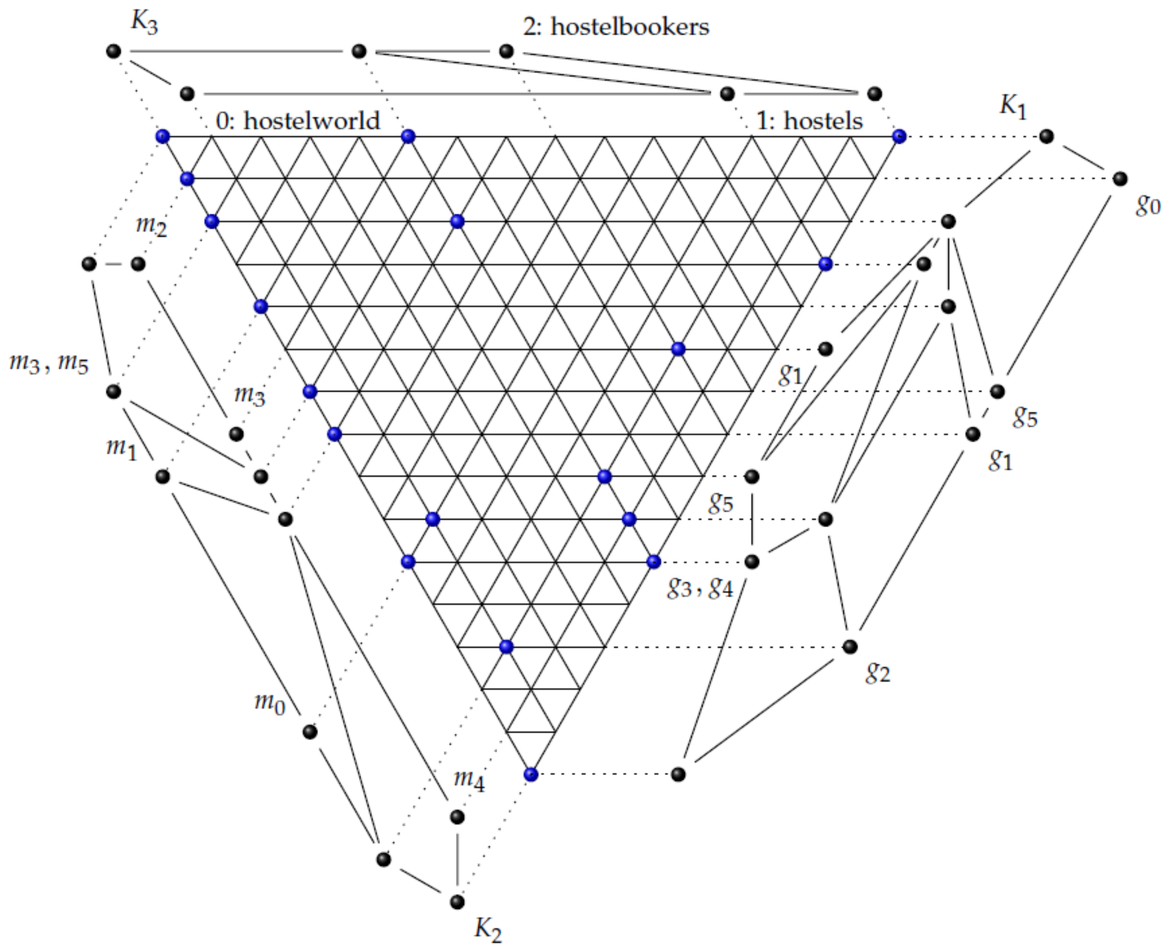
Knowledge Discovery

Lecture 9: Triadic FCA

Triadic FCA (3FCA)

- Target: triadic data sets either having a natural 3D structure or mapped on 3D in order to find relevant knowledge structures.
- Mine after relevant knowledge
- Cluster relevant information into triconcepts
- Represent triconcepts in a form which is suitable for decision making
- Analyse the structure of the triconcepts set in order to find more relevant knowledge structures.

Trilattice example (4)



Source: Factor Analysis of triadic data - C.V.Glodeanu

Disadvantages of the trilattice representation

- no automated tool to generate it
- difficult to read and to navigate in
- the underlying structure of the triconcepts cannot be read from the trilattice

Proposed solution

- A local navigation paradigm in the tricontexts:
- one can get from a triconcept to another if it is **reachable**
- once chosen a triconcept you can see all possible next steps

Derived contexts

Definition (Derived contexts)

Every triadic context (K_1, K_2, K_3, Y) *gives rise to the following projected dyadic contexts:*
$\mathbb{K}^{(1)} := (K_1, K_2 \times K_3, Y^{(1)})$ *with* $gY^{(1)}(m,b) :\Leftrightarrow (g,m,b) \in Y$,
$\mathbb{K}^{(2)} := (K_2, K_1 \times K_3, Y^{(2)})$ *with* $mY^{(2)}(g,b) :\Leftrightarrow (g,m,b) \in Y$,
$\mathbb{K}^{(3)} := (K_3, K_1 \times K_2, Y^{(3)})$ *with* $bY^{(3)}(g,m) :\Leftrightarrow (g,m,b) \in Y$.
For $\{i,j,k\} = \{1,2,3\}$ *and* $A_k \subseteq K_k$, *we define* $\mathbb{K}^{(ij)}_{A_k} := (K_i, K_j, Y^{(ij)}_{A_k})$, *where* $(a_i, a_j) \in Y^{(ij)}_{A_k}$ *if and only if* $(a_i, a_j, a_k) \in Y$ *for all* $a_k \in A_k$.

- $\mathbb{K}^{(i)}$ = **flattened** versions of the triadic context, obtained by putting the "slices" of (K_1, K_2, K_3, Y) side by side.
- $\mathbb{K}^{(ij)}_{A_k}$ = the intersection of all those slices that correspond to elements of A_k.

Derivation operators

Definition ((*i*)-derivation operators)

For $\{i, j, k\} = \{1, 2, 3\}$ *with* $j < k$ *and for* $X \subseteq K_i$ *and* $Z \subseteq K_j \times K_k$ *the* (i)*-derivation operators are defined by:*

$X \mapsto X^{(i)} := \{(a_j, a_k) \in K_j \times K_k \mid (a_i, a_j, a_k) \in Y \text{ for all } a_i \in X\}.$

$Z \mapsto Z^{(i)} := \{a_i \in K_i \mid (a_i, a_j, a_k) \in Y \text{ for all } (a_j, a_k) \in Z\}.$

Definition ((*i*, *j*, X_k)-derivation operators)

For $\{i, j, k\} = \{1, 2, 3\}$ *and* $X_i \subseteq K_i, X_j \subseteq K_j, X_k \subseteq K_k$*, the* (i, j, X_k)*-derivation operators are defined by*

$X_i \mapsto X_i^{(i,j,X_k)} := \{a_j \in K_j \mid (a_i, a_j, a_k) \in Y \text{ for all } (a_i, a_k) \in X_i \times X_k\}$

$X_j \mapsto X_j^{(i,j,X_k)} := \{a_i \in K_i \mid (a_i, a_j, a_k) \in Y \text{ for all } (a_j, a_k) \in X_i \times X_k\}.$

Recall triadic concepts

- triadic concepts are maximal cuboids of incidences

Definition

A **triadic concept** *(short:* **triconcept***) of* $\mathbb{K} := (K_1, K_2, K_3, Y)$ *is a triple* (A_1, A_2, A_3) *with* $A_i \subseteq K_i$ *for* $i \in \{1, 2, 3\}$ *and* $A_i = (A_j \times A_k)^{(i)}$ *for every* $\{i, j, k\} = \{1, 2, 3\}$ *with* $j < k$. *The sets* A_1, A_2, *and* A_3 *are called* **extent, intent,** *and* **modus** *of the triadic concept, respectively. We let* $\mathfrak{T}(\mathbb{K})$ *denote the set of all triadic concepts of* $\mathbb{K}$.

Exercise assignement

Let **K**=(K1, K2, K3, Y) be a triadic context, where

K1 = {1,2,3,4},

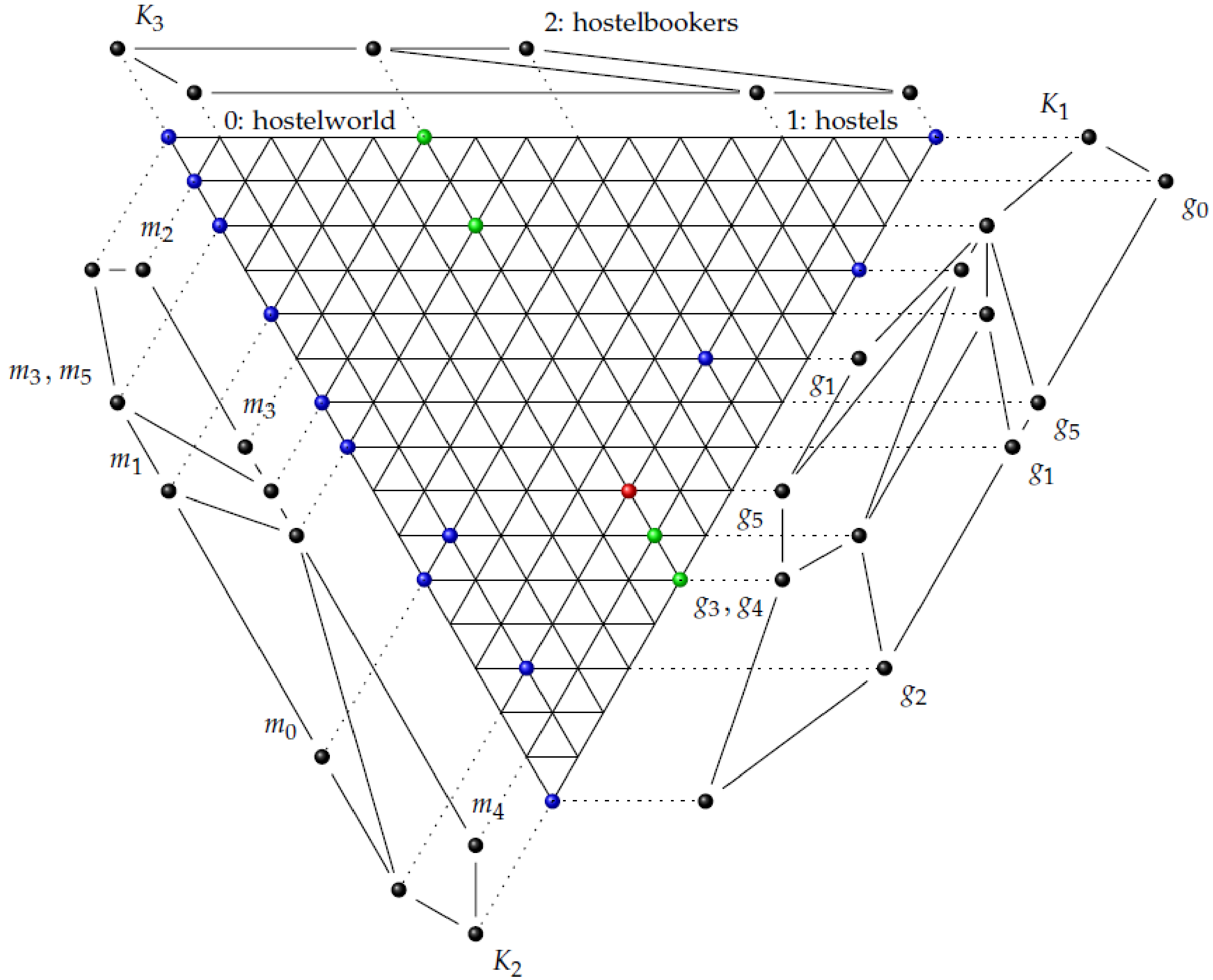
K2 = {a,b,c,d},

K3 = {α, β, γ}, and

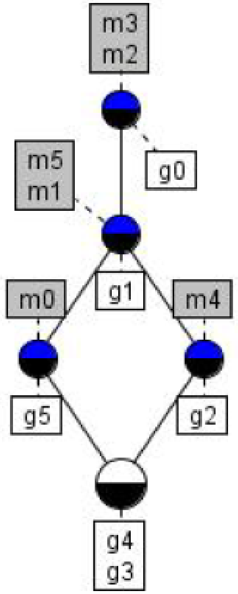
Y={(1,a,β), (1,a,γ), (1,b,α), (1,b,β), (1,c,α), (1,c,β), (2,b,α), (2,b,β), (2,a,β),(2,a,γ),(3,a,β),(3,a,γ),(3,d,α),(4,d,β),(4,a,β),(4,a,γ),(4,b,α), (4,b,β)

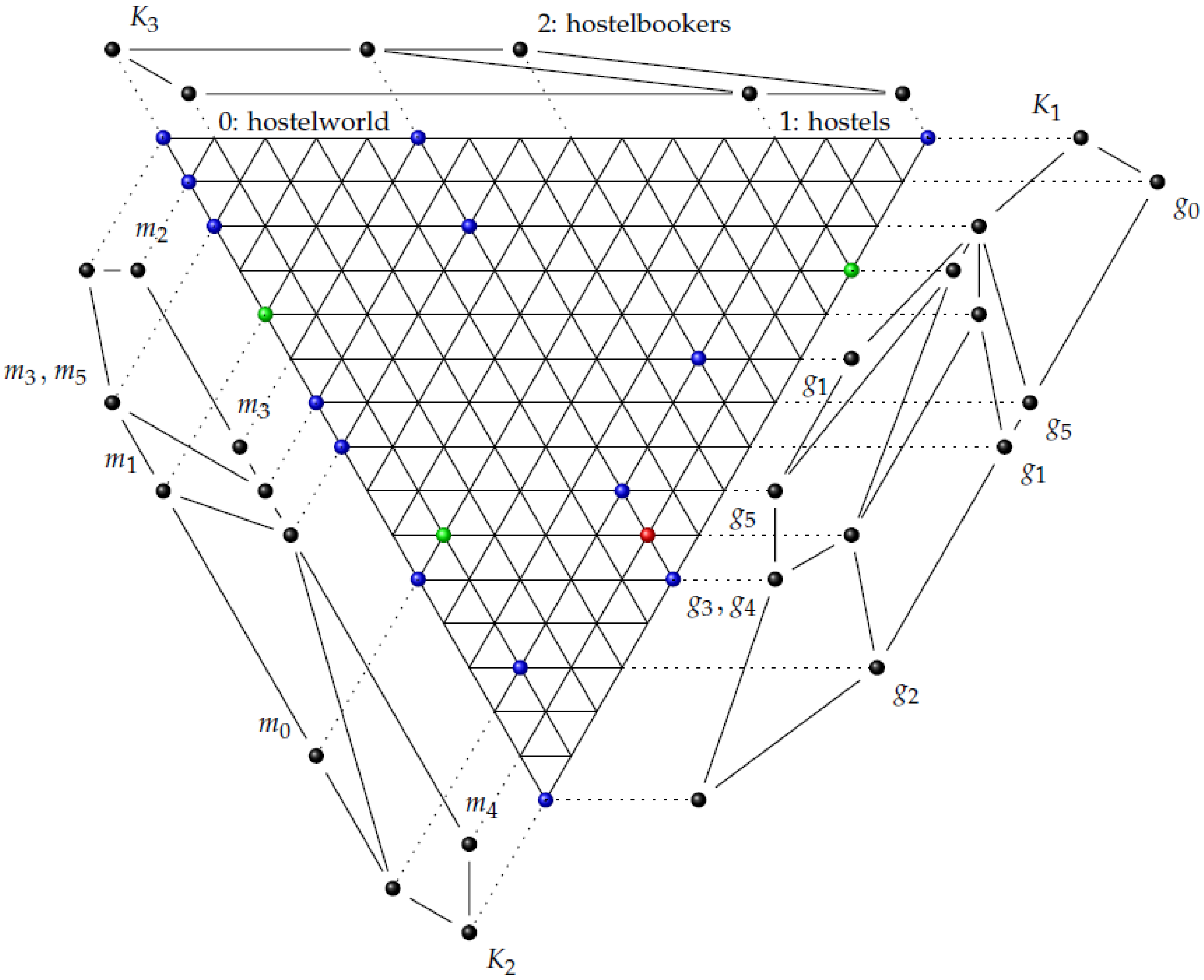
1. Give all three slice representations of **K**.
2. Compute three different triconcepts of **K**.
3. For a triconcept (A1, A2, A3) compute the projections $\mathbf{K}^{ij}_{Ak}$.

Navigation example



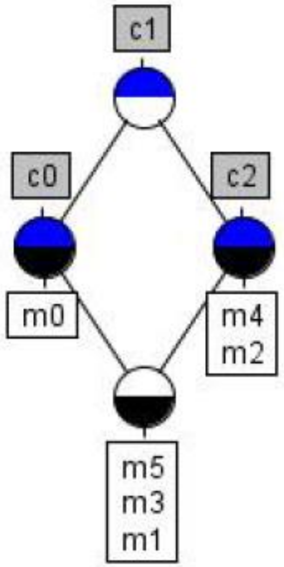
Choose $T := (\{g_3, g_4, g_5\}, \{m_0, m_1, m_2, m_3, m_5\}, \{c_1, c_2\})$ as the triconcept wherefrom local navigation starts and consider perspective 3 (i.e., modus). By projecting along $\{c_1, c_2\}$, we obtain the following concept lattice.
Triconcept T corresponds to the leftmost dyadic concept.





K_3
2: hostelbookers
0: hostelworld
1: hostels
K_1
g_0
m_2
m_3, m_5
m_3
g_1
g_5
g_1
m_1
g_5
g_3, g_4
g_2
m_0
m_4
K_2

By choosing the rightmost concept of this lattice corresponding to the triconcept $(\{g_2, g_3, g_4\}, \{m_2, m_3, m_4\}, \{c_1, c_2\})$ as a next step and the first perspective the reachable triconcepts are:



Reachable triconcepts

Definition

For (A_1, A_2, A_3) *and* (B_1, B_2, B_3) *triadic concepts, we say that* (B_1, B_2, B_3) *is* directly reachable *from* (A_1, A_2, A_3) using perspective (1) *and we write* $(A_1, A_2, A_3) \prec_1 (B_1, B_2, B_3)$ *if and only if* $(B_2, B_3) \in \mathfrak{B}(\mathbb{K}_{A_1}^{(23)})$. *Analogously, we can define direct reachability using perspectives* (2) *and* (3).

We say that (B_1, B_2, B_3) *is* directly reachable *from* (A_1, A_2, A_3) *if it is directly reachable using at least one of the three perspectives, that is, formally*

$(A_1, A_2, A_3) \prec (B_1, B_2, B_3) :\Leftrightarrow [(A_1, A_2, A_3) \prec_1 (B_1, B_2, B_3)] \vee [(A_1, A_2, A_3) \prec_2 (B_1, B_2, B_3)] \vee [(A_1, A_2, A_3) \prec_3 (B_1, B_2, B_3)]$.

Remark

$\mathbb{K}_{A_k}^{(ij)} := (K_i, K_j, Y_{A_k}^{(ij)})$, *where* $(a_i, a_j) \in Y_{A_k}^{(ij)}$ *if and only if* $(a_i, a_j, a_k) \in Y$ *for all* $a_k \in A_k$.

Mutually reachable triconcepts

Proposition

Let $(A_1, A_2, A_3), (B_1, B_2, B_3)$ be two triconcepts. If $A_i = B_i$ for an $i \in \{1, 2, 3\}$ then $(A_1, A_2, A_3) \prec_i (B_1, B_2, B_3)$ and $(B_1, B_2, B_3) \prec_i (A_1, A_2, A_3)$.

Reachability for triconcepts

Definition

We define the **reachability relation** *between two triconcepts as being the transitive closure of the direct reachability relation* $\prec$. *We denote this relation by* $\lhd$.

Definition

The equivalence class of a triconcept (A_1, A_2, A_3) *with respect to the preorder* $\lhd$ *on* $\mathfrak{T}(\mathbb{K})$ *will be called a* **reachability cluster** *and denoted by* $[(A_1, A_2, A_3)]$.

Obtaining the clusters of triconcepts using a graph

- Consider a directed graph with triconcepts as nodes and with directed edges between the triconcepts for which the reachability relation holds
- The strongly connected components of the graph represent the **clusters** of triconcepts.

More than two clusters

α	1	2	3
a	×		
b			
c			

β	1	2	3
a	×		
b		×	
c			

γ	1	2	3
a	×		
b		×	
c			×

The triconcepts are partitioned in clusters the following way:
$C_1 = \{(\{c\}, \{3\}, \{\gamma\})\}$
$C_2 = \{(\{b\}, \{2\}, \{\beta, \gamma\})\}$
$C_3 = \{(\{a\}, \{1\}, \{\alpha, \beta, \gamma\}), (\{a, b, c\}, \{1, 2, 3\}, \emptyset),$
$(\{a, b, c\}, \emptyset, \{\alpha, \beta, \gamma\}), (\emptyset, \{1, 2, 3\}, \{\alpha, \beta, \gamma\})\}$

$\mathbf{C_1 < C_2 < C_3}$

Remark

The triconcepts $(\{c\}, \{3\}, \{\gamma\})$ *and* $(\{b\}, \{2\}, \{\beta, \gamma\})$ *have disjoint extents and intents, but* $(\{c\}, \{3\}, \{\gamma\}) \prec_3 (\{b\}, \{2\}, \{\beta, \gamma\})$.

Reachability in composed tricontexts

There is a way of composing several tricontexts such that the reachability clusters of the composed tricontext coincide with the union of the reachability clusters of the constituents, except for the greatest cluster.

Definition

Given tricontexts $\mathbb{K}_1 := (K_1^1, K_2^1, K_3^1, Y^1), \ldots, \mathbb{K}_n := (K_1^n, K_2^n, K_3^n, Y^n)$, *with* K_i^j *and* K_i^k *being disjoint for all* $j \neq k$ *and all* $i \in \{1, 2, 3\}$, *their* composition $\mathbb{K}_1 \uplus \ldots \uplus \mathbb{K}_n$ *is the tricontext* $\mathbb{K} := (K_1, K_2, K_3, Y)$ *with* $K_i := \bigcup_{k=1}^{n} K_i^k$ *and* $Y := \bigcup_{k=1}^{n} Y^k$.

Size of the tricontext vs. Number of reachability clusters

We can find qubic tricontexts (i.e., $|K_1| = |K_2| = |K_3| = n$), where the number of clusters equals $n + 1$:

Proposition

Let $\mathbb{K} = (K_1, K_2, K_3, Y)$ be a tricontext of size $n \times n \times n$ with $K_1 = \{k_i^1 \mid 1 \leq i \leq n\}$, $K_2 = \{k_i^2 \mid 1 \leq i \leq n\}$, $K_3 = \{k_i^3 \mid 1 \leq i \leq n\}$. Let the relation Y be the spatial main diagonal of the tricontext, meaning that a triple $(k_i^1, k_j^2, k_l^3) \in Y$ iff $i = j = k$. Then there are n+1 clusters, n minimal clusters and the maximal cluster.

The number of minimal clusters is not bounded by the dimension of the context

Consider the following $6 \times 6 \times 4$ tricontext $\mathbb{K}_{664}$.

α	1	2	3	4	5	6
a	×					
b		×				
c			×			
d						
e						
f						

β	1	2	3	4	5	6
a	×					
b						
c						
d				×		
e					×	
f						

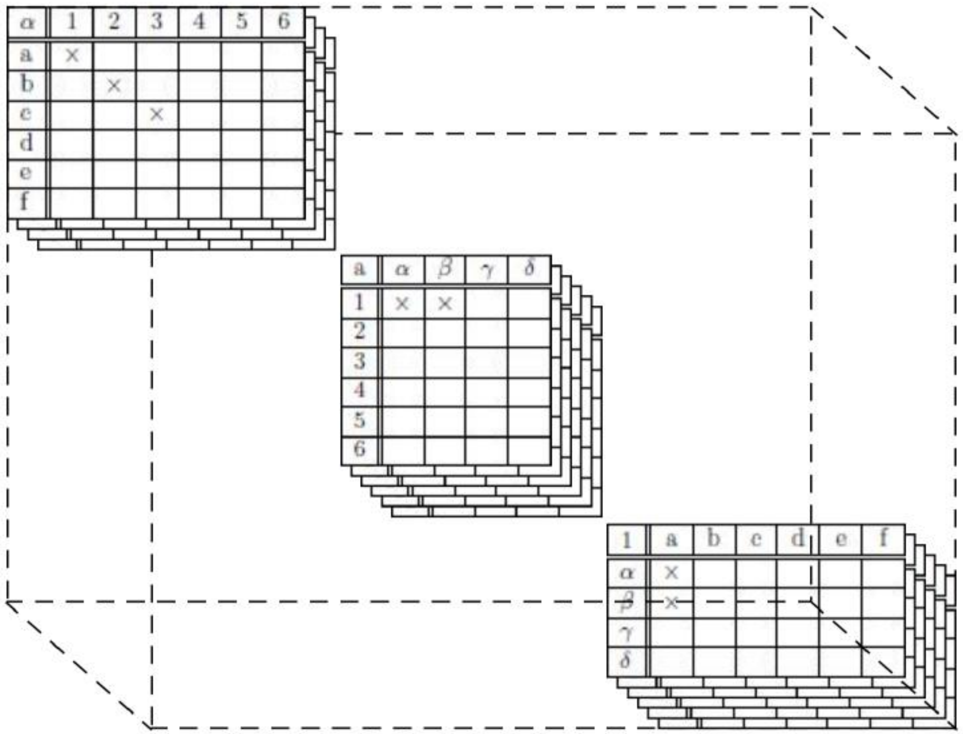
γ	1	2	3	4	5	6
a						
b		×				
c						
d				×		
e						
f						×

δ	1	2	3	4	5	6
a						
b						
c			×			
d						
e					×	
f						×

Besides the maximal cluster, we have six minimal ones which are all singletons consisting of the following triconcepts, respectively:

$C_1 := (\{a\}, \{1\}, \{\alpha, \beta\}), \quad C_2 := (\{b\}, \{2\}, \{\alpha, \gamma\}),$
$C_3 := (\{c\}, \{3\}, \{\alpha, \delta\}), \quad C_4 := (\{d\}, \{4\}, \{\beta, \gamma\}),$
$C_5 := (\{e\}, \{5\}, \{\beta, \delta\}), \quad C_6 := (\{f\}, \{6\}, \{\gamma, \delta\}).$

Consider the $16 \times 16 \times 16$ context built by composing the previous context with its rotations: $\mathbb{K}_{16^3} := \mathbb{K}_{664} \uplus \mathbb{K}_{646} \uplus \mathbb{K}_{466}$.



We can prove that $\mathbb{K}_{16^3}$ has 19 clusters, the maximal one and $6 + 6 + 6 = 18$ minimal ones, respectively.

Exercise assignment

- Use FCA TOOLS BUNDLE and the local navigation feature for triadic data sets.

The context of reachable triconcepts

Definition

Let $\mathbb{K} := (K_1, K_2, K_3, Y)$ *be a triadic context. We denote by* $\mathfrak{T}(\mathbb{K})$ *the set of all triconcepts. Then, the formal context of reachable triconcepts is defined as* $\mathbb{K}_{\triangleleft} := (\mathfrak{T}(\mathbb{K}), \mathfrak{T}(\mathbb{K}), \triangleleft)$.

Remark

We denote with $\mathbb{C}$ the set of clusters of triconcepts from $\mathbb{K}$ and with $\mathbb{I}$ the following set: $\mathbb{I} = \{A \cap B | (A, B) \in \mathbb{K}_{\triangleleft}\} \setminus \{\emptyset\}$,

Assumption

$\mathbb{I} = \mathbb{C}$

Structure of the clusters

Proposition

Let $(A, B) \in \mathbb{K}_\triangleleft$ be a concept and denote by $C := A \cap B$. If $C \neq \emptyset$, then C is a set of mutually reachable concepts, i.e., $C \times C$ is a rectangle of crosses in $\mathbb{K}_\triangleleft$.

Proof of $\mathbb{I} = \mathbb{C}$:

- Every intersection $A \cap B$ of a concept $(A, B) \in \mathbb{K}_\triangleleft$ is part of a cluster
- An intersection is also maximal $\Rightarrow$ an intersection is equal to a cluster
- For every cluster C there is a concept with the intersection equal to the cluster: $(C^{\triangleleft\triangleleft}, C^{\triangleleft})$

Conclusion

The set of clusters covers only a part of a lattice, the concepts $(A, B) \in \mathbb{K}_\triangleleft$ which have a nonempty intersection.

OPEN QUESTIONS

- What is the depth of the graph of triconcepts?
- What is the maximal number of clusters that a tricontext can have in relation with its dimensions? Current upper bound is the number of triconcepts, which is exponential.
- What is the number of minimal clusters that a tricontext can have in relation with its dimensions?
- If we use the reachability relation as a navigation paradigma, which is a suitable starting point? Obviously the starting point should be in a minimal cluster, but which is the right minimal cluster?

Clarified contexts

- **dyadic case**: a context is called **clarified** if there are no identical rows and columns
- **triadic case**: a context is called **clarified** if there are no identical slices
- reason: since a triconcept (A_1, A_2, A_3) is a maximal triple of triadic incidences, removing identical slices in the tricontext does not alter the structure of triconcepts

Definition

A triadic context (K_1, K_2, K_3, Y) *is* **clarified** *if for every* $i \in \{1, 2, 3\}$ *and every* $u, v \in K_i$*, from* $u^{(i)} = v^{(i)}$ *follows* $u = v$.

Advantages of reduction

- **context reduction** has no effect on the conceptual structure
- **reducible objects and attributes** can be written as combinations of other objects and attributes
- we can reduce the data quantity without losing information

Reduced dyadic context

Definition

In a dyadic context, an element is **reducible** *if and only if the row/column of that element is the intersection of some rows/columns.*

Definition

A clarified context (G, M, I) *is called*

- **object reduced** *if it has no reducible objects*
- **attribute reduced** *if it has no reducible attributes*

Reduced triadic context

Definition

Let $\mathbb{K} = (K_1, K_2, K_3, Y)$ be a tricontext and $a_i \in K_i$, $i = 1, 2, 3$. Then the element a_i is **reducible** *if and only if there exist a subset $X \subseteq K_i$ with $Y_X^{(jk)} = Y_{\{a_i\}}^{(jk)}$, where*

$$Y_X^{(jk)} := \{(b_j, b_k) \in K_j \times K_k \mid \forall b_i \in X.\ (b_i, b_j, b_k) \in Y\},$$ *for* $\{i, j, k\} = \{1, 2, 3\}$.

The definition states that a_i is reducible if and only if the slice of a_i is the intersection of some slices corresponding to the elements of a certain subset $X \subseteq K_i$

Definition

A clarified context (K_1, K_2, K_3, Y) is called

- **object reduced** *if it has no reducible objects*
- **attribute reduced** *if it has no reducible attributes*
- **condition reduced** *if it has no reducible conditions*

- if we switch the role of the objects with that of the attributes we look at the context $(M, G, I^{-1}) \Rightarrow$ former attribute concepts are now object concepts
- using this observation, an equivalent definition of the reduced context can be given using joins as follows

Definition

A clarified context (G, M, I) *is called* **row reduced** *if every object concept is* $\vee$*-irreducible and* **column reduced** *if every attribute concept is* $\wedge$*-irreducible.*

Definition

A clarified tricontext (K_1, K_2, K_3, Y) *is called* **object reduced** *if every object concept from the context* $(K_1, K_2 \times K_3, Y^{(1)})$ *is* $\vee$*-irreducible,* **attribute reduced** *if every object concept from the context* $(K_2, K_3 \times K_1, Y^{(2)})$ *is* $\vee$*-irreducible, and* **condition reduced** *if every object concept from the context* $(K_3, K_1 \times K_2, Y^{(3)})$ *is* $\vee$*-irreducible.*

Reduction example

g_1	b_1	b_2	b_3
m_1	×	×	×
m_2			
m_3		×	

g_2	b_1	b_2	b_3
m_1		×	×
m_2			×
m_3	×		

g_3	b_1	b_2	b_3
m_1		×	×
m_2			
m_3			

- non-trivial triconcepts of this context are:
  - $(\{g_1\}, \{m_1\}, \{b_1, b_2, b_3\})$
  - $(\{g_2\}, \{m_3\}, \{b_1\})$
  - $(\{g_1, g_2, g_3\}, \{m_1\}, \{b_2, b_3\})$
  - $(\{g_1\}, \{m_1, m_3\}, \{b_2\})$
  - $(\{g_2\}, \{m_1, m_2\}, \{b_3\})$
- we observe the slice corresponding to g_3 is the intersection of the slides corresponding to g_1 and g_2
- by reducing object g_3, the number of triconcepts remains unchanged and the trilattice will be the same

A different characterisation of reducible elements

Definition

Let $\mathbb{K} := (K_1, K_2, K_3, Y)$ be a tricontext. For $g \in K_1, m \in K_2, b \in K_3$ we define the following relations, where $\swarrow$ is the arrow relation from dyadic FCA:

- $(g, m, b) \in \triangleleft \Leftrightarrow g \swarrow (m, b)$
- $(g, m, b) \in \triangle \Leftrightarrow m \swarrow (g, b)$
- $(g, m, b) \in \triangleright \Leftrightarrow b \swarrow (g, m)$
- $(g, m, b) \in \ast \Leftrightarrow (g, m, b) \in \triangleleft$ *and* $(g, m, b) \in \triangle$, *and* $(g, m, b) \in \triangleright$

With this notation, an element $a_i \in K_i$ will be reducible if and only if its corresponding slice, i.e., $(K_j, K_k, Y_{a_i}^{(jk)})$ does not contain the triadic arrow $\ast$

R. Wille:

Definition 25. If (G, M, I) is a context, $g \in G$ an object, and $m \in M$ an attribute, we write

$$g \swarrow m \quad :\Longleftrightarrow \quad \begin{cases} g \not{I} m \text{ and} \\ \text{if } g' \subseteq h' \text{ and } g' \neq h', \text{ then } hIm, \end{cases}$$

$$g \nearrow m \quad :\Longleftrightarrow \quad \begin{cases} g \not{I} m \text{ and} \\ \text{if } m' \subseteq n' \text{ and } m' \neq n', \text{ then } gIn, \end{cases}$$

$$g \swarrow\!\!\!\!\nearrow m \quad :\Longleftrightarrow \quad g \swarrow m \text{ and } g \nearrow m.$$

◇

Exercise assignment

- Use ConExp to practice reducing using arrow relations in formal contexts.

Object concepts

In the triadic case, an object concept should be defined as a set of triconcepts

Definition

Let $\mathbb{K} := (K_1, K_2, K_3, Y)$ be a tricontext, $g \in K_1$, $m \in K_2$, and $b \in K_3$ be objects, attributes, and conditions, respectively. The **object concept** *of g is defined as $\gamma^{\Delta}(g) := \{(A_1, A_2, A_3) \in \mathfrak{T}(\mathbb{K}) \mid A_1 = g^{(1)(1)}\}$, where $(\cdot)^{(i)}$ is the derivation operator g in $\mathbb{K}^{(i)}$, $i \in \{1, 2, 3\}$. Similar, the* **attribute concept** *of m is defined as $\mu^{\Delta}(m) := \{(A_1, A_2, A_3) \in \mathfrak{T}(\mathbb{K}) \mid A_2 = m^{(2)(2)}\}$, while the* **condition concept** *of b is defined as $\beta^{\Delta}(b) := \{(A_1, A_2, A_3) \in \mathfrak{T}(\mathbb{K}) \mid A_3 = b^{(3)(3)}\}$.*

Experiments

- the cancer registry database, in its original form, contains **25** attributes for each patient, including an identification number, for example *Tumor sequence*, *Topography*, *Morphology*, *Behavior*, *Basis of diagnosis*, *Differentiation degree*, *Surgery*, *Radiotherapy*, *Hormonal Therapy*, *Curative Surgery*, *Curative Chemotherapy*
- to prepare the data for a triadic interpretation, the knowledge management suite ToscanaJ and Toscana2Trias, a triadic extension developed at Babes-Bolyai University Cluj-Napoca, have been used
- Toscana2Trias uses the TRIAS algorithm developed by Jäschke in order to compute the triconcepts of the data represented as a triadic context

Example

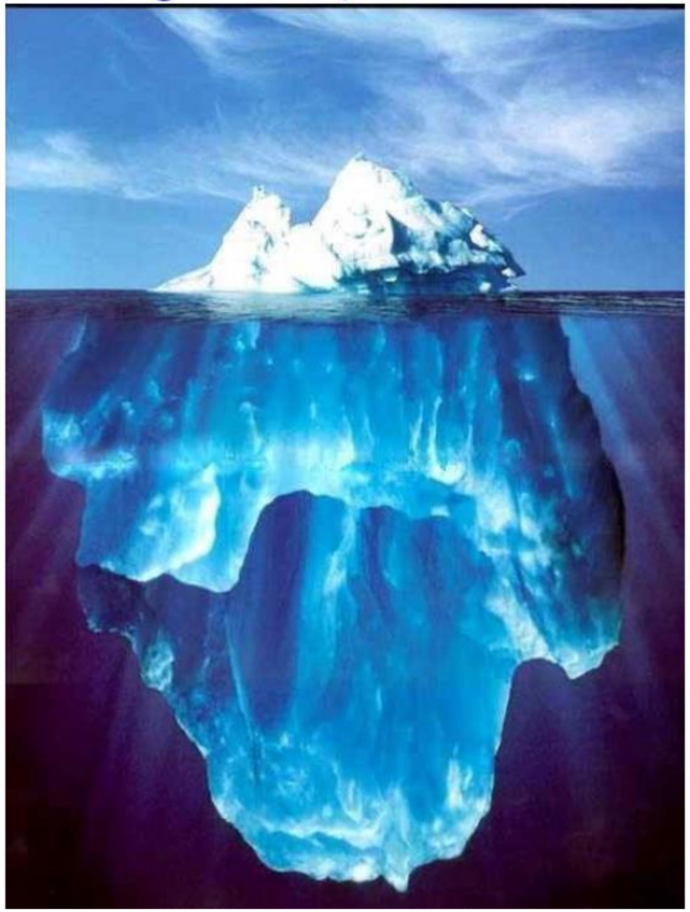
- we have selected a number of **4686 objects, 11 attributes** (all 8 degrees of certainty in the oncological decision process, in-situs carcinoma and tumor sequence 1, i.e., just one tumor) and **3 conditions** ($Gender = Male$, $age < 59$, and $survival > 30\ months$)
- this selection of a tricontext with **4686 objects**, **11 attributes** and **3 conditions** contains **44545 tuples** (crosses in the tricontext) and **63 triconcepts**
- the clarified tricontext contains **61 objects**, **11 attributes** and **3 conditions**
- the reduced context contains **23 objects**, **4 attributes** and **3 conditions**, resulting in a relation with **77 tuples**

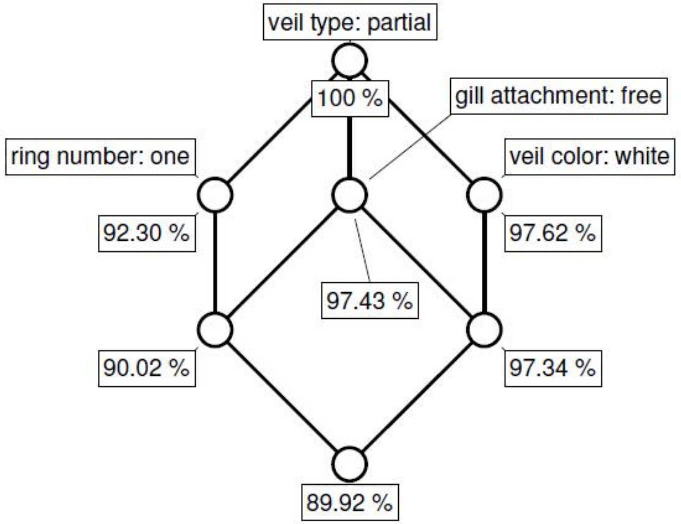
Conclusion

Role of reduction for triadic FCA:

- eliminates redundant information
- increases efficiency in determining the underlying conceptual structure
- reducible objects (or attributes, conditions) may give important clues about logical dependencies in the data

Iceberg Concept Lattices





The seven most general concepts (for minsupp = 85%) of the 32086 concepts of the mushroom database (http://kdd.ics.uci.edu/).

Iceberg Concept Lattices: Support

Def.: The *support* of a set $X \subseteq M$ of attributes is defined as

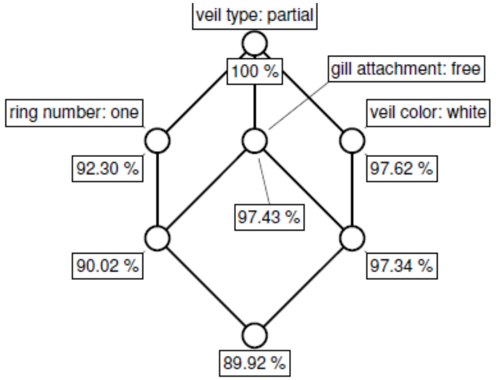
$$\text{supp}(X) := \frac{|X'|}{|G|}$$

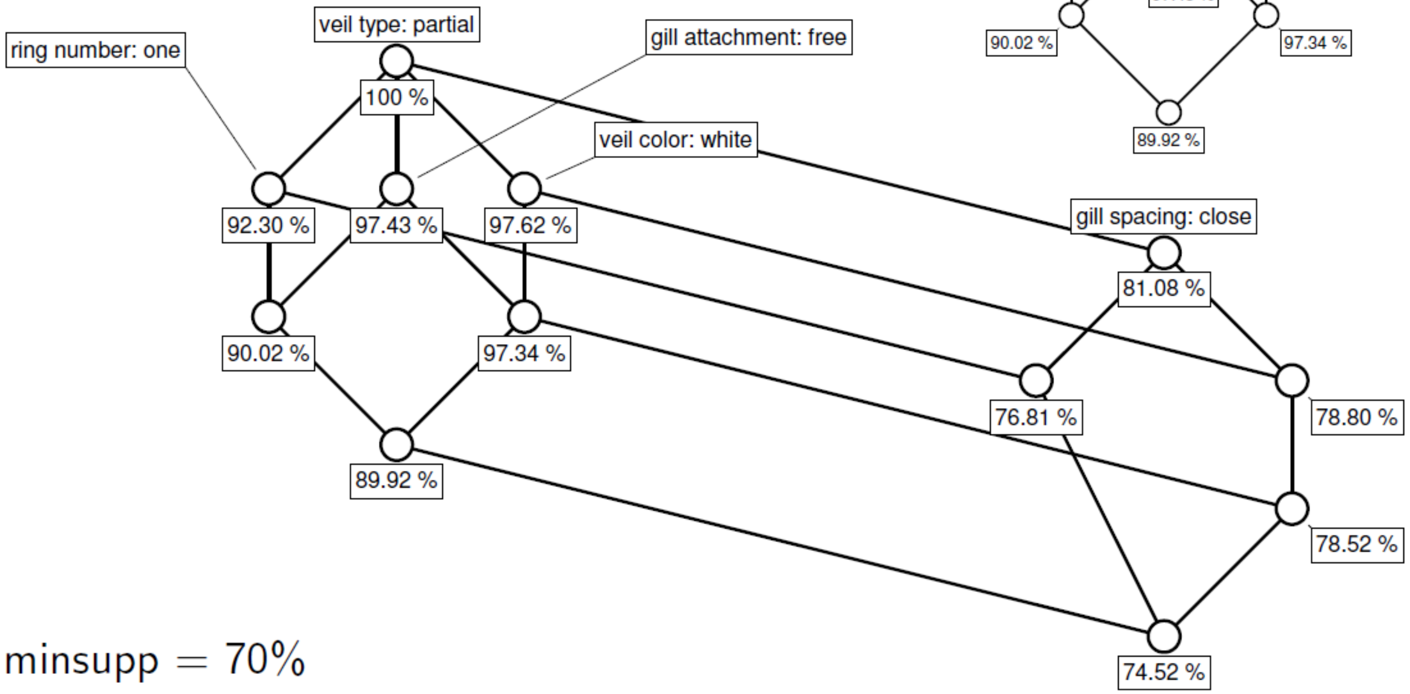
Def.: The *iceberg concept lattice* of a formal context (G, M, I) for a given minimal support value $minsupp$ is the set

$$\{(A, B) \in \underline{\mathfrak{B}}(G, M, I) \mid \text{supp}(B) \geqslant minsupp\}$$

Iceberg Concept Lattices

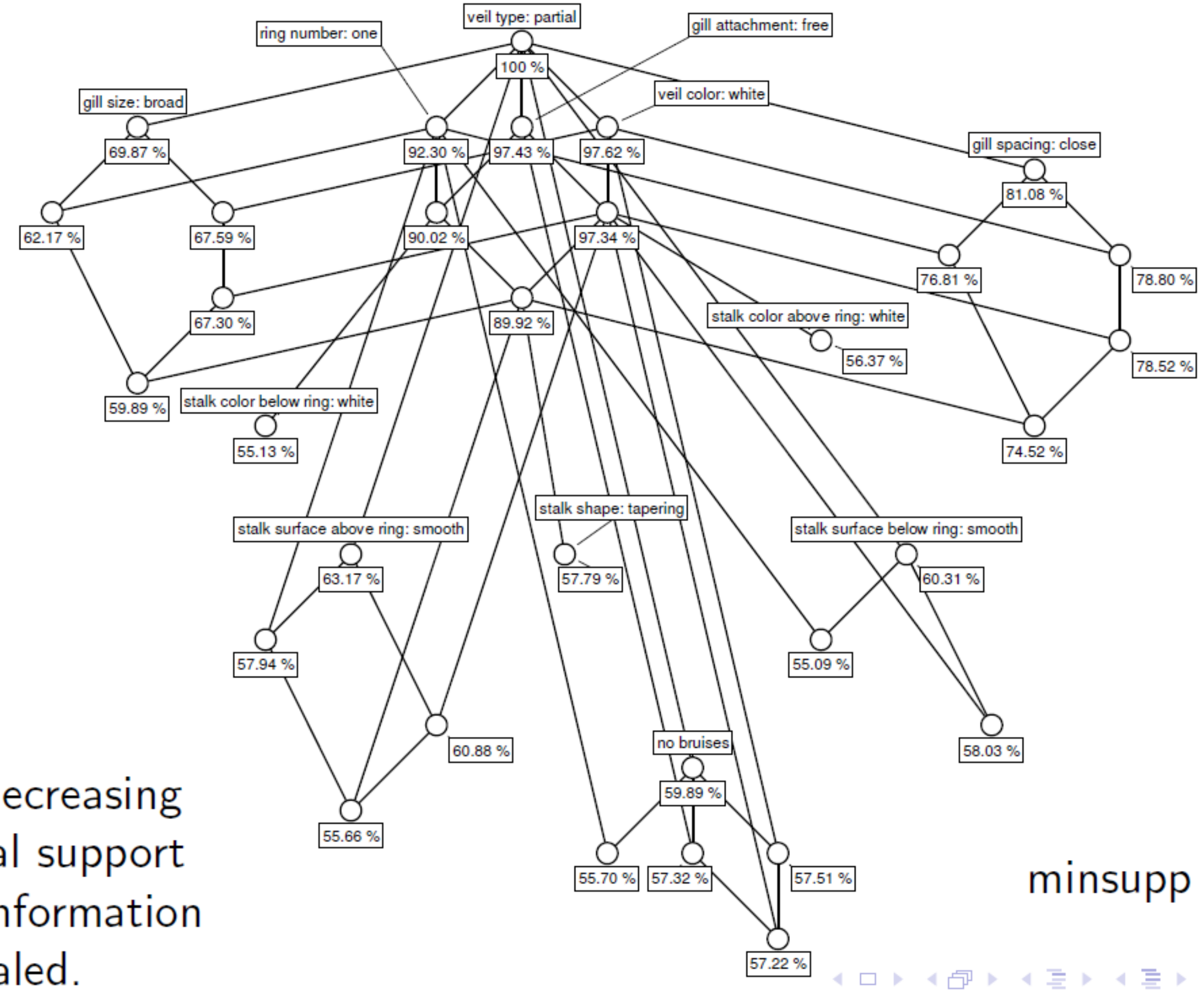
$\text{minsupp} = 85\%$





$\text{minsupp} = 70\%$

Iceberg Concept Lattices



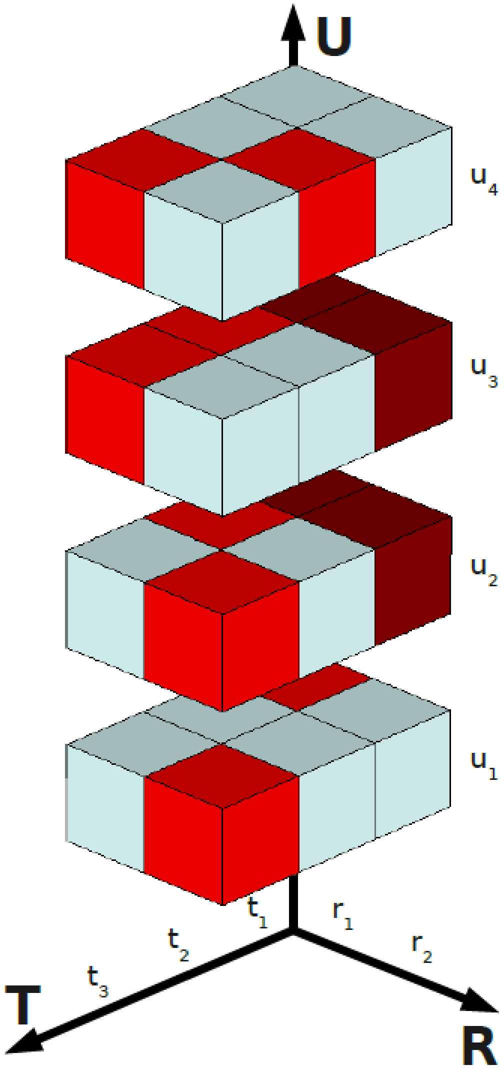
With decreasing minimal support more information is revealed.

minsupp $= 55\%$

Iceberg Tri-Lattices

- Given support constraints τ_u, τ_t, τ_r : tri-concept (A, B, C) *frequent* $:\Leftrightarrow\ |A| \geqslant \tau_u$, $|B| \geqslant \tau_t$, and $|C| \geqslant \tau_r$

  → *iceberg tri-lattice*



Computing Tri-Concepts

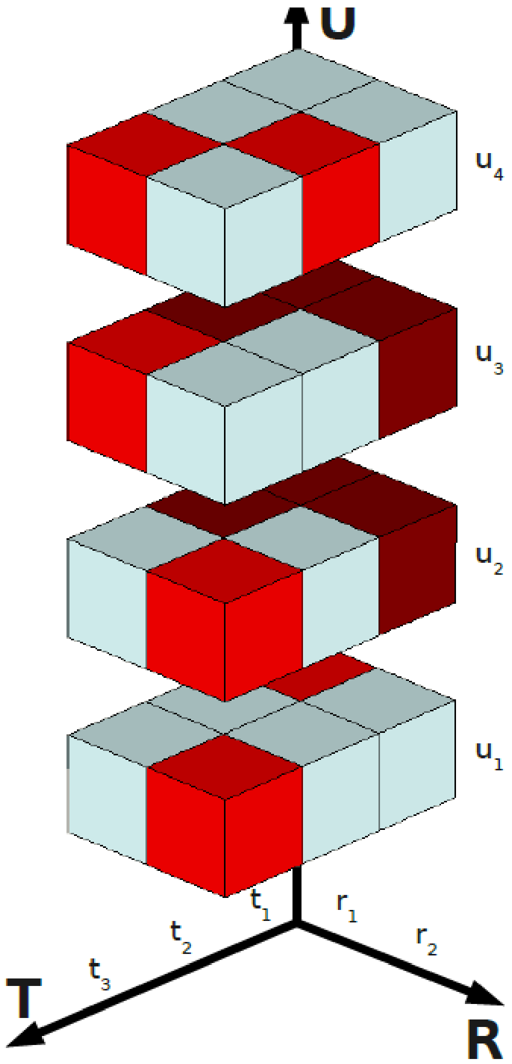
- Given
  - sets U, T, R
  - ternary relation $Y \subseteq U \times T \times R$
  - support constraints τ_u, τ_t, τ_r
- Find (A, B, C) with
  - $A \subseteq U$, $B \subseteq T$, $C \subseteq R$
  - $|A| \geqslant \tau_u$, $|B| \geqslant \tau_t$, $|C| \geqslant \tau_r$
  - $A \times B \times C \subseteq Y$
  - such that none of the sets A, B or C can be enlarged without violating the former condition

Computing Tri-Concepts

computes the iceberg tri-lattice of a triadic formal context

Algorithm

- Let $\tilde{Y} := \{(u, (t, r)) \mid (u, t, r) \in Y\}$
- Loop: Find (frequent) concepts $(\mathbf{A}, I)$ in $(U, T \times R, \tilde{Y})$



In the example:

$(A, I) =$

$(\{u_2, u_3\},\ \{(t_1, r_1), (t_1, r_2), (t_2, r_1)\})$

Computing Tri-Concepts

computes the iceberg tri-lattice of a triadic formal context

Algorithm

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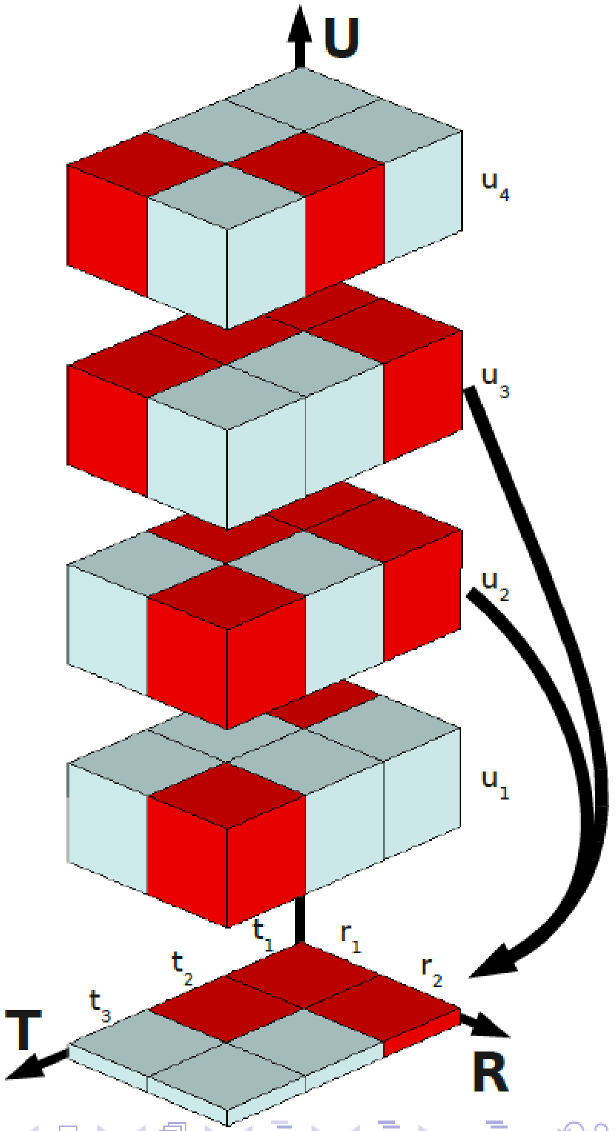
  - Loop: Find (frequent) concepts $(\mathbf{B}, \mathbf{C})$ in (T, R, I)

In the example:
$(T, R, I) = (T, R, \{(t_1, r_1), (t_1, r_2), (t_2, r_1)\})$



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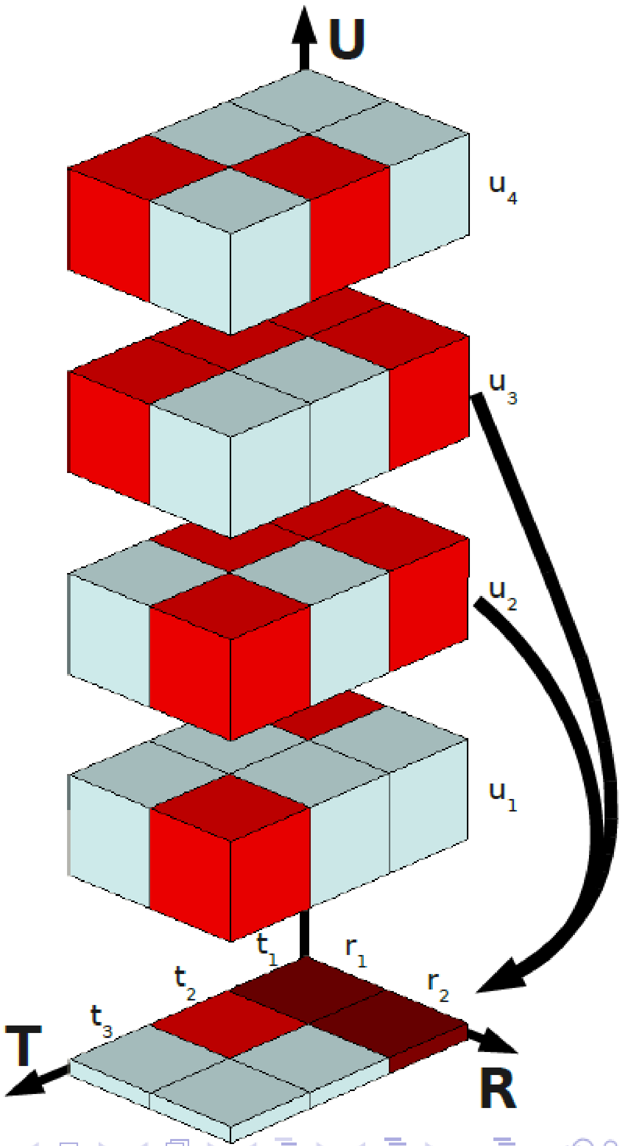
In the example:
$(B, C) = (\{t_1\}, \{r_1, r_2\})$



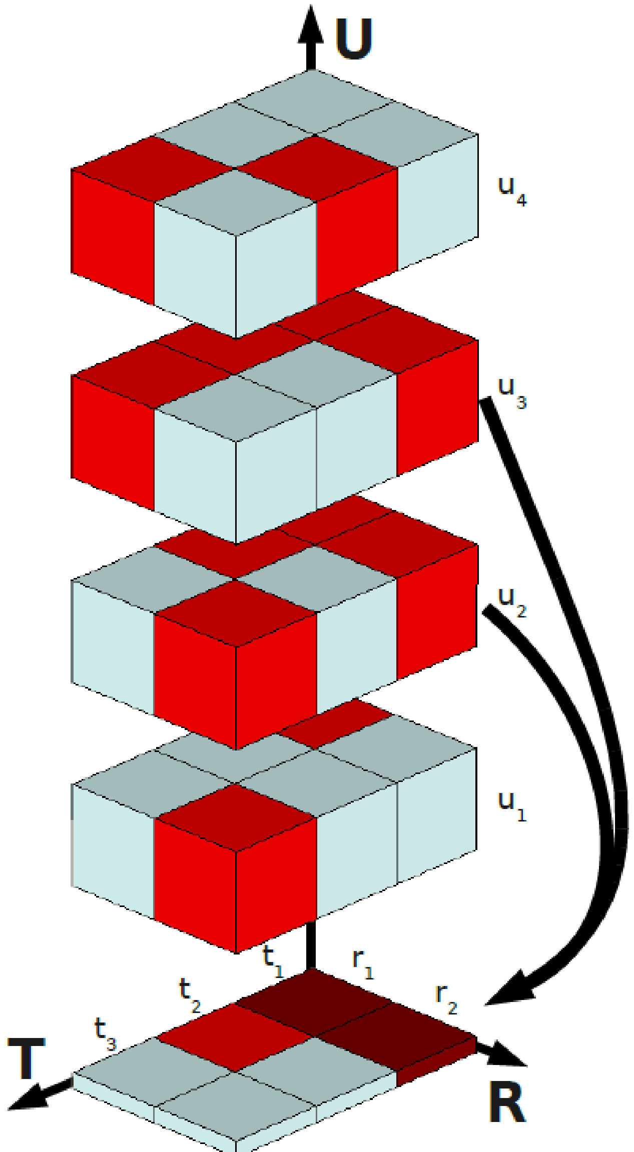
Computing Tri-Concepts

computes the iceberg tri-lattice of a triadic formal context

Algorithm

- Let $\tilde{Y} := \{(u,(t,r)) \mid (u,t,r) \in Y\}$
- Loop: Find (frequent) concepts $(\mathbf{A}, I)$ in $(U, T \times R, \tilde{Y})$
  - Loop: Find (frequent) concepts $(\mathbf{B}, \mathbf{C})$ in (T, R, I)
    - If $\mathbf{A} = (\mathbf{B} \times \mathbf{C})^{\tilde{Y}}$, then output $(\mathbf{A}, \mathbf{B}, \mathbf{C})$

In the example:
$(B \times C)^{\tilde{Y}} = (\{t_1\} \times \{r_1, r_2\})^{\tilde{Y}}$



Computing Tri-Concepts

computes the iceberg tri-lattice of a triadic formal context

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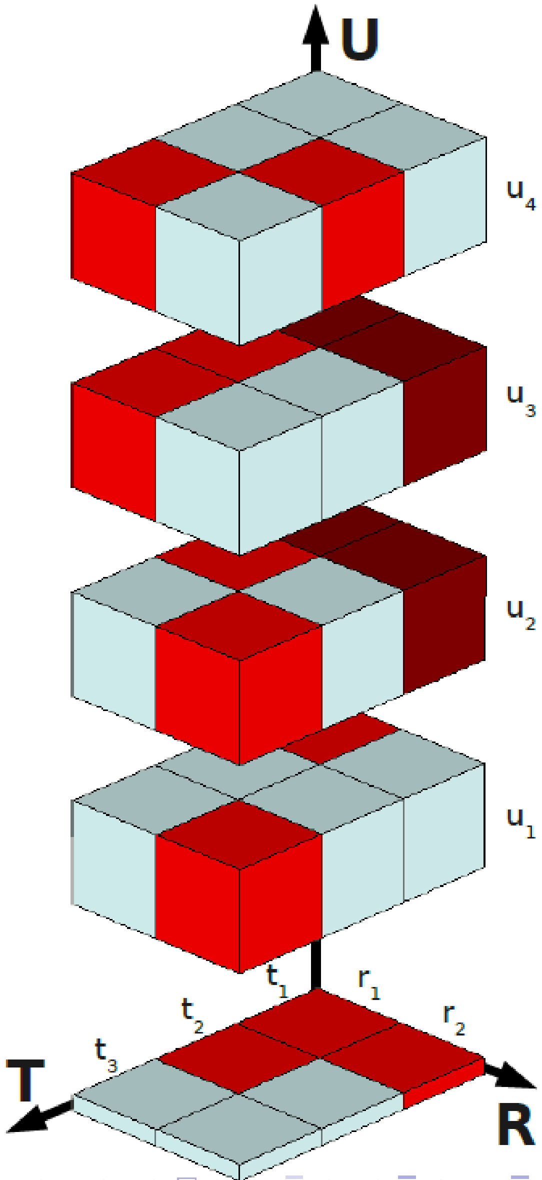
In the example:

$$(B \times C)^{\tilde{Y}} = (\{t_1\} \times \{r_1, r_2\})^{\tilde{Y}} = \{u_2, u_3\} = A$$



Computing Tri-Concepts

computes the iceberg tri-lattice of a triadic formal context

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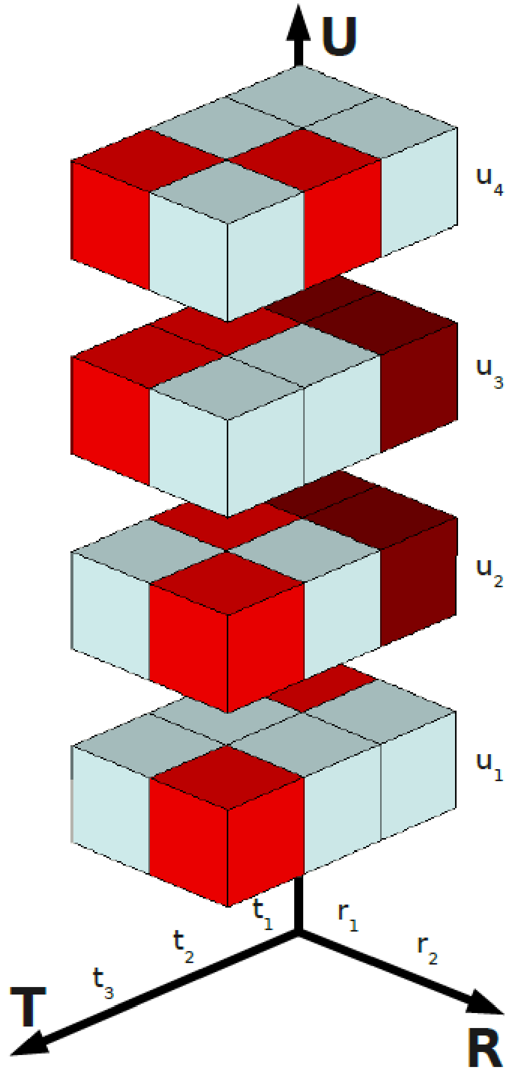
In the example:
$(A, B, C) = (\{u_2, u_3\}, \{t_1\}, \{r_1, r_2\})$



Computing Tri-Concepts

Require: $U, T, R, Y, \tau_u, \tau_t, \tau_r$

$\tilde{Y} := \{(u,(t,r)) \mid (u,t,r) \in Y\}$

$(A, I) :=$ *FirstFrequentConcept*$((U, T \times R, \tilde{Y}), \tau_u)$

repeat

  if $|I| \geqslant \tau_t \cdot \tau_r$ **then**

    $(B, C) :=$ *FirstFrequentConcept*$((T, R, I), \tau_t)$

    repeat

      if $|C| \geqslant \tau_r$ **then**

        if $A = (B \times C)^{\tilde{Y}}$ **then**

          print A,B,C

        end if

      end if

    until not *NextFrequentConcept*$((B, C), (T, R, I), \tau_t)$

  end if

until not *NextFrequentConcept*$((A, I), (U, T \times R, \tilde{Y}), \tau_u)$

Computing Tri-Concepts

The *FirstFrequentConcept* method:

Require: $(G, M, I), \tau$

$A := \emptyset^I$

$B := A^I$

if $|A| < \tau$ **then**

  NextFrequentConcept$((A, B), (G, M, I), \tau)$

end if

return (A, B)

Computing Tri-Concepts

the *NextFrequentConcept* method:

Require: $(A, B), (G, M, I), \tau$

i := *max*(M)
while *defined*(i) **do**
    $A := (B \bullet i)^I$
    if $|A| \geqslant \tau$ **then**
        $D := A^I$
        if $B <_i D$ **then**
            $B := D$
            return true
        end if
    end if
    $i :=$ *max*$(M \backslash B \cap \{1, \ldots, i-1\})$
end while
return false

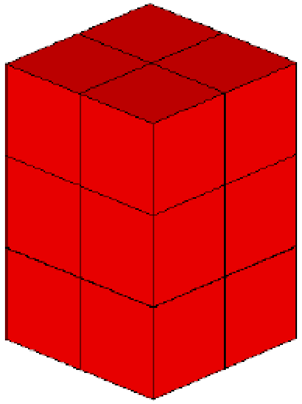
Evaluation

BibSonomy Dataset:

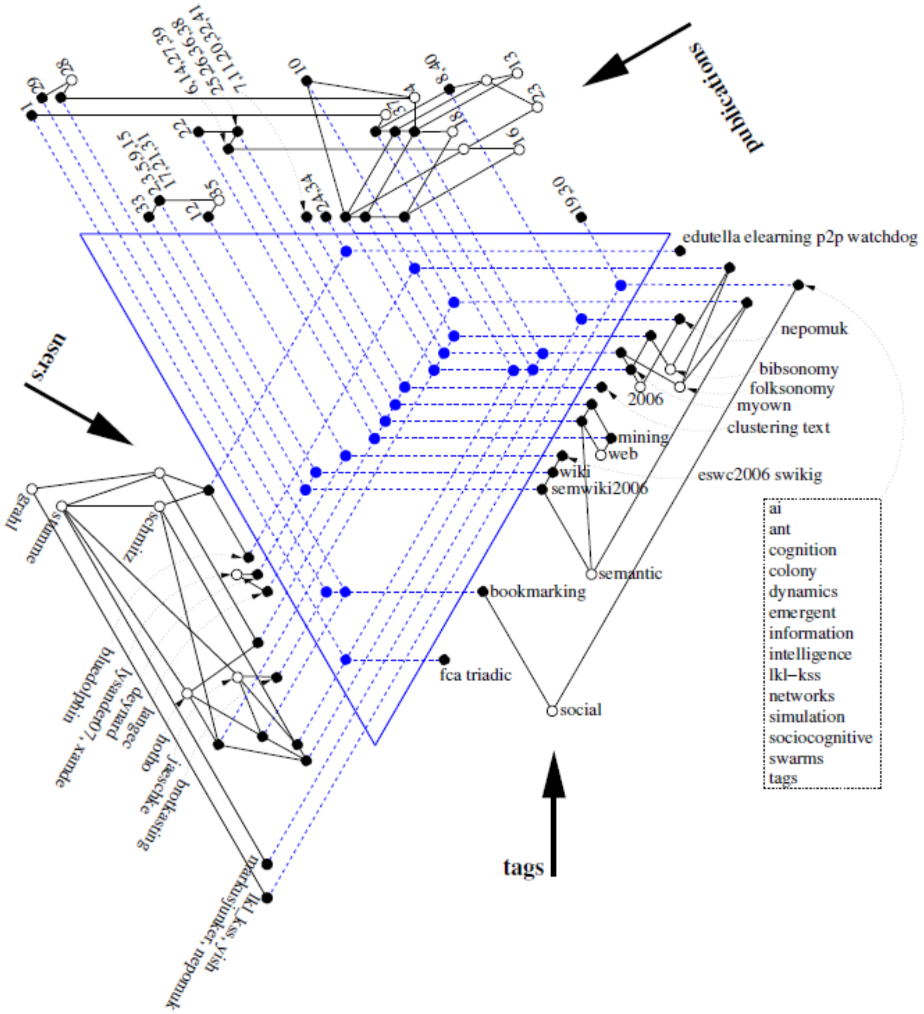
- all publication records until November 23rd, 2006
- removed: DBLP, posts with the tag "imported"
- $|U| = 262$, $|T| = 5\,954$, $|R| = 11\,101$, $|Y| = 44\,944$

Result:

- 13 992 tri-concepts (75 minutes on a 2 GHz PC)
- with support constraints $\tau_u = 3$, $\tau_t = 2$, $\tau_r = 2$:
  - 21 tri-concepts
  - contain 41 publications, 15 users and 36 tags

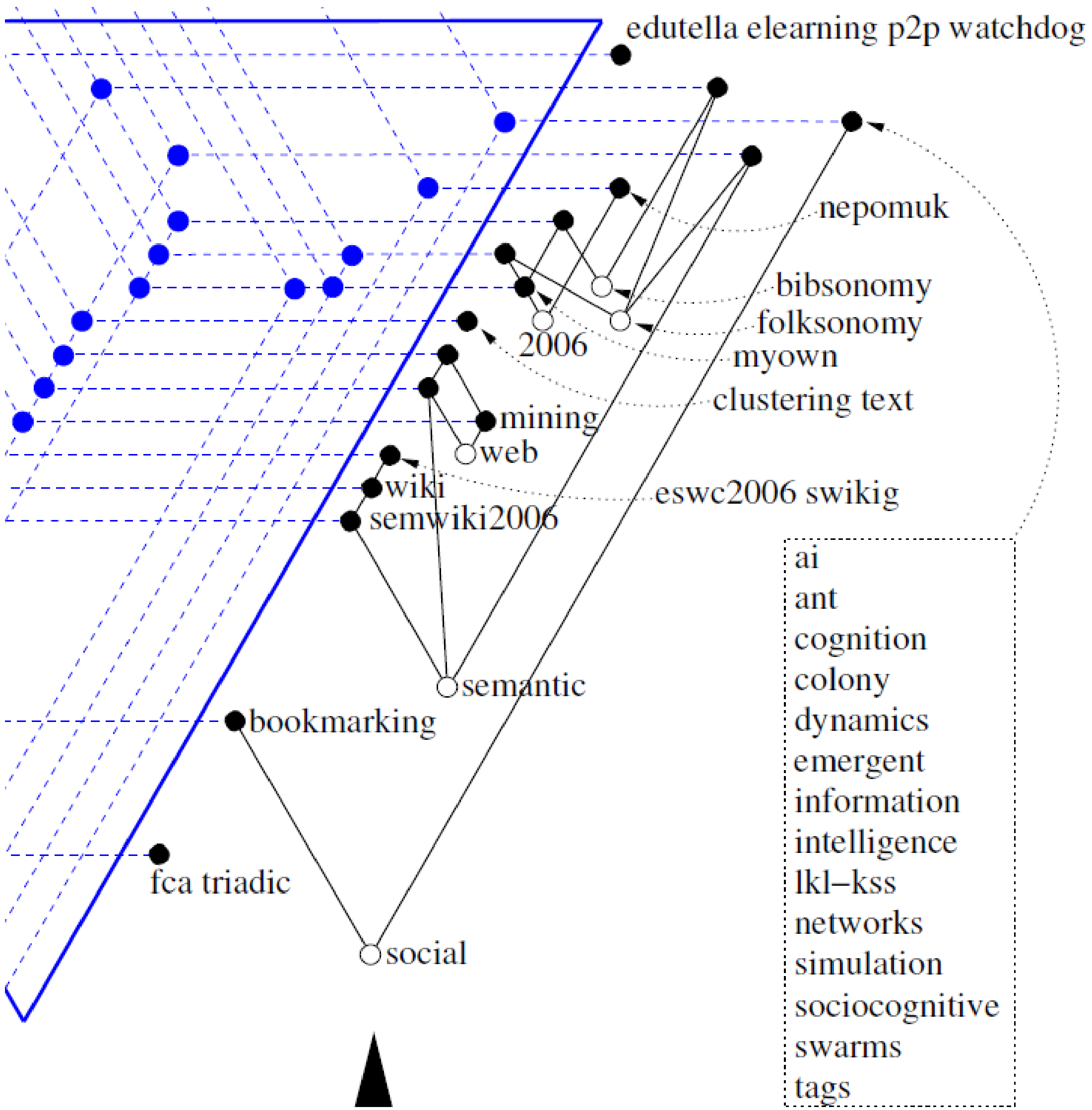


Evaluation



visualisation of the iceberg tri-lattice for $\tau_u = 3$, $\tau_t = 2$, $\tau_r = 2$

Evaluation



two topical groups:

- semantic
- social

semantic further divided:

- wiki
- web
- folksonomy

Neighborhoods

The visualization of tri-lattices is . . .

- at the moment manual work,
- time-intensive and pretty complicated,
- or even impossible (cf. *tetrahedron condition* and *Thomson condition*).

Thus: easier visualization option desireable

Neighborhoods

Idea:

- We regard tri-concepts as nodes in a graph.
- We connect two tri-concepts with an edge, when they contain the same tags, users, or resources.

More formally:

- Two tri-concepts (A_1, A_2, A_3) and (B_1, B_2, B_3) are *neighbors*, if for an $i \in \{1, 2, 3\}$ it holds $A_i = B_i$.
- neighbor relation $\sim \subseteq (\underline{\mathfrak{B}}(\mathbb{F}) \times \underline{\mathfrak{B}}(\mathbb{F}))$
- The *neighborhood graph* then is $(\underline{\mathfrak{B}}(\mathbb{F}), \sim)$.

Neighborhoods

neighborhood graph for the tri-concept

$(\{jaeschke, schmitz, stumme\}, \{fca, triadic\}, \{1, 37\})$