THE SQUARE-FREE $\mathbb{Z}$-MODULES

The notion of a *square-free module* first appeared in [2].
"A module M is called square-free if it contains no non-zero isomorphic submodules A and B with $A \cap B = 0$."

The square-free Abelian groups (i.e., $\mathbb{Z}$-modules), are easy to characterize.

Theorem 1. *An Abelian group G is square-free (as $\mathbb{Z}$-module) iff its torsion-free rank $r_0(G) \leq 1$ and for every prime p, all its p-ranks $r_p(G) \leq 1$.*

References

[1] S. H. Mohamed *Internal exchange rings*. Algebra and its applications, Contemp. Math., vol. 419, Amer. Math. Soc., Providence, RI, 2006, pp. 213-218.

[2] Saad H. Mohamed and Bruno J. Müller, *Continuous and Discrete Modules*, LMS Lecture Note Series 147, Cambridge University Press, 1990.

[3] Mazurek, P. P. Nielsen, M. L Ziembowski *Commuting idempotents, square-free modules and the exchange property*. J. of Algebra **444** (2015), 52-80.

[4] P. P. Nielsen *Square-free modules with the exchange property*. J. of Algebra **323** (7) (2010), 1993-2001.