

# Negative of nil-clean is clean

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It is well-known that a nil-clean element may not be clean. However

**Exercise 1** *The negative of a nil-clean element in any ring with identity is clean.*

**Solution 2** *Suppose  $a = e + t$  with idempotent  $e$  and nilpotent  $t$ . Then  $-a = (1 - e) - (1 + t)$  is clean (with idempotent  $1 - e$  and unit  $1 + t$ ).*