Correction to "Abelian groups with C2"

March 8, 2025

We accepted the following statement from [1] without criticism:

Proposition 2.9. Let R be a commutative domain and M be a torsion-free module. Then M is a direct-injective module if and only if M is a divisible module.

And even more specific: $\mathbb{Z}$ is not direct-injective (equivalently C2) as a $\mathbb{Z}$-module.

However, every indecomposable module, not having proper direct summands, is trivially (vacuously) C2.

Therefore all indecomposable $\mathbb{Z}$-modules (that is, Abelian groups) are C2. In particular, $\mathbb{Z}$ is C2 and as it is well-known, there are plenty of torsion-free indecomposable (Abelian) groups, which this way, all are C2.

Consequently, our Corollary 3 should be revised as follows: *a torsion-free group is C2 iff it is indecomposable or divisible.*

We thank Radoslav Dimitric, for drawing our attention to this oversight.

References

[1] S. K. Maurya, A. J. Gupta *A note on direct-injective modules*. Palest. J. Math. **8** (1) (2019) 235-241.