Abelian groups whose subgroups are endomorphic kernels

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In [1], the *Abelian groups whose subgroups are endomorphic images* were determined and in [2] the dual problem was solved: the groups G whose factor groups can be embedded in G were found (property denoted Q).

A natural question would be to find the *groups whose subgroups are endomorphic kernels*.

However, these are exactly the groups with the property Q above.

Proof. One way is obvious: if every subgroup is a kernel of endomorphism, let $H = \ker(f)$ with $f \in \mathrm{End}(G)$. Then by Noether isomorphism theorem $G/H = G/\ker(f) \cong f(G)$ which is a subgroup of G.

Conversely, for a subgroup H, denote by $p_H : G \longrightarrow G/H$ the canonic projection. If G has the property Q, there is an embedding $\mu : G/H \longrightarrow G$. Then $H = \ker(p_H) = \ker(\mu \circ p_H)$ as desired.

References

[1] L. Fuchs, A. Kertesz, T. Szele *On abelian groups whose subgroups are endomorphic images*. Acta Sci. Math. Szeged, **16** (1955), 77–88.

[2] L. Fuchs, A. Kertesz, T. Szele *On abelian groups in which every homomorphic image can be imbedded*, Acta Math. Acad. Sci. Hung., **7** (1956), 467–475.