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Beyond the Identity Theorem: Reconstructing Analytic Functions from a Sequence of Values

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Abstract

This talk was inspired by the following remark in Ahlfors's *Complex Analysis* [1, p. 127]: "*We can also say that an analytic function is uniquely determined by its values on any set with an accumulation point in the region of analyticity. This does not mean that we know of any way in which the values of the function can be computed.*"

Motivated by this observation, we consider the following reconstruction problem. Let f be analytic on $\Omega = \mathbb{C}\setminus S$, where S is a known set of nonremovable isolated singularities, with no finite accumulation point. Suppose that the values $f(a_1), f(a_2), \ldots$ are known, where $(a_n)_{n\geq 1} \subset \Omega$ is a sequence of distinct points converging to $a \in \Omega$. Although analytic continuation through successive local Taylor expansions is possible in principle, it may require several intermediate expansions and become cumbersome, particularly on large or multiply connected domains.

As an alternative, we present a constructive method based on rational interpolation with poles prescribed at selected singularities. Each rational approximant is computed from finitely many sampled values using only finitely many algebraic operations, without evaluating contour integrals. A single limiting process then recovers the function on suitable regions.

An exact Hermite-type remainder formula yields geometric convergence criteria expressed in terms of a potential function. We investigate circular domains and generalized lemniscates, obtaining explicit conditions under which the reconstruction converges. We also show how oversampling can improve the convergence regions. By progressively including additional singularities and using an appropriate diagonal sequence of rational interpolants, we obtain locally uniform reconstruction throughout Ω, even when S is infinite.

Moreover, the polynomial and partial-fraction decomposition of the rational interpolants allows us to reconstruct the canonical regular and individual singular Cauchy components on suitable multiply connected domains, again through algebraic expressions involving only the sampled values.

Finally, we discuss connections with classical interpolation theory, Hermite's remainder formula, Padé-type rational approximation, and Sobieszek's Newton–Laurent interpolation series.

This is a joint work with Nicolae R. Pascu.

References

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