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The Ahlfors-Weill reflection on convex domains and Nehari quasidisks

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Abstract

The estimate

$$\mathrm{Re}\{a_2 f\} > -\frac{1}{2}$$

derived for convex mappings in [FMR], is interpreted here in terms of the Ahlfors-Weill reflection to show that for such domains Ω, the mediatrix of the segment $[w, \mathcal{R}(w)]$ joining a point $w \in \Omega$ and its reflection $\mathcal{R}(w)$ lies always outside Ω. In particular, the midpoint of the segment is also outside Ω. We determine the extremal cases when the midpoint can be on the boundary $\partial\Omega$. The normalization to a Möbius equivalent mapping with vanishing second coefficient given by

$$f^* = \frac{f}{1 + a_2 f}$$

leads to important distinctions between bounded and unbounded domains. Finally, we derive a new characterization of Nehari quasidisks in terms of upper and lower bounds of the form

$$c_1 \rho(w, \partial\Omega) \leq \rho(\mathcal{R}(w), \Omega) \leq c_2 \rho(w, \partial\Omega)\ ,\ w \in \Omega,$$

where ρ stands for the spherical metric and c_1, c_2 are constants depending on the quasidisk.

[FMR] R. Fournier, J. Ma and S. Ruscheweyh, *Convex univalent functions and omitted values*, Approximation Theory: In Memory of AK Varma (2021), p. 225.