Workshop dedicated to the memory of Professor Gabriela Kohr
(6th edition)
Geometric Function Theory in Several Complex Variables and Complex Banach Spaces
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About cyclicity in weighted Dirichlet spaces

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Abstract

Given a bounded operator T acting on a Hilbert space H, a vector $x \in H$ is called *cyclic* if $\{p(T)x : \ p \text{ polynomial}\}$ is dense in H. This talk is concerned with the case when T is the forward shift on a class of Dirichlet-type spaces on the unit disc and pertains also to the case of several complex variables, where the definition of cyclicity is modified accordingly. It is quite obvious that in such Hilbert spaces of analytic functions, cyclic elements cannot have zeros in the domain in question, but sufficient conditions for cyclicity involves more subtle ideas. For example, one could consider the appropriate capacity of the set of zeros of the radial limits of the function in question a path which leads to the very natural, but still unsolved Brown-Shields conjecture. Alternatively, the lack of "boundary zeros" of the function f is also implied if $\frac{1}{f}$, or $\log f$ belongs to the space and it turns out that these conditions imply cyclicity for such Dirichlet-type spaces. They can also be considered for Dirichlet-type spaces on the unit ball. The aim of the talk is to give an account about some recent results in this direction. The material is based on joint work with S. Richter, and also with K.M. Perfekt, S. Richter, C. Sundberg, and J. Sunkes.