Faculty of Mathematics and Computer Science
Babeş-Bolyai University of Cluj-Napoca

Workshop dedicated to the memory of
Professor Gabriela Kohr
(4th edition)

Geometric Function Theory in Higher Dimensions and Complex Banach Spaces

Book of Abstracts

November 29 – December 1, 2024
Cluj-Napoca, Romania

Invited Speakers

- **Zoltán Balogh** - University of Bern, Switzerland
- **Lucian Beznea** - Institute of Mathematics of the Romanian Academy and Politehnica University of Bucharest, Romania
- **Filippo Bracci** - Tor Vergata University, Rome, Italy
- **Teodor Bulboacă** - Babeş-Bolyai University, Cluj-Napoca, Romania
- **Dan Coman** - Syracuse University, New York, USA
- **Mihai Cristea** - University of Bucharest, Romania
- **Paula Curt** - Babeş-Bolyai University, Cluj-Napoca, Romania
- **Tamás Darvas** - University of Maryland, USA
- **Santiago Diaz-Madrigal** - University of Seville, Spain
- **Mark Elin** - Braude College, Karmiel, Israel
- **Aurelian Gheondea** - Bilkent University, Ankara, Turkey and Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania
- **Anatoly Golberg** - Holon Institute of Technology, Holon, Israel
- **Ian Graham** - University of Toronto, Canada
- **Pavel Gumenyuk** - Polytechnical University of Milan, Italy
- **Hidetaka Hamada** - Kyushu Sangyo University, Fukuoka, Japan
- **Takahiro Hasebe** - Hokkaido University, Sapporo, Japan
- **Tatsuhiro Honda** - Senshu University, Tokyo, Japan
- **Mihai Iancu** - Babeş-Bolyai University, Cluj-Napoca, Romania
- **Fiana Jacobzon** - Braude College, Karmiel, Israel
- **Cezar Joiţa** - Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania

- **Massimo Lanza de Cristoforis** - University of Padova, Italy
- **Sergey E. Mikhailov** - Brunel University London, UK
- **Dorina Mitrea** - Baylor University, Texas, USA
- **Irina Mitrea** - Temple University, Philadelphia, USA
- **Marius Mitrea** - Baylor University, Texas, USA
- **Takuya Murayama** - Kyushu University, Fukuoka, Japan
- **Camil Muscalu** - Cornell University, New York, USA
- **Victor Nistor** - Universite Lorraine, Metz, France
- **Mihai Pascu** - Transilvania University of Braşov, Romania
- **Cornel Pintea** - Babeş-Bolyai University, Cluj-Napoca, Romania
- **Eric Schippers** - University of Manitoba, Canada
- **Alexandru Tămăşan** - University of Central Florida, USA
- **Wolfgang L. Wendland** - University of Stuttgart, Institute for Applied Analysis and Numerical Simulation & SIMTECH, Germany

Main organizer

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Logarithmic Sobolev inequalities on Euclidean submanifolds: sharpness, rigidity and applications

Zoltán Balogh
University of Bern, Switzerland

Abstract

We provide Michael–Simon-type L^p-logarithmic-Sobolev inequalities on complete, not necessarily compact n-dimensional submanifolds Σ of the Euclidean space $\mathbb{R}^{n+m}$. Our estimate is sharp, and it involves the mean curvature of Σ. Equality can only occur if and only if Σ is isometric to the Euclidean space $\mathbb{R}^n$ and the extremizer is a Gaussian. Applications are provided to sharp hypercontractivity estimates of Hopf–Lax semigroups on submanifolds. This is a joint work with Alexandru Kristály.

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Quasi-regular Dirichlet forms and multiplicative semigroups on L^p

Lucian Beznea
Simion Stoilow Institute of Mathematics of the Romanian Academy and Politehnica University of Bucharest, Romania

Abstract

We show that a Markovian multiplicative semigroup on an L^p space is generated by a continuous flow. However, an extension of the state space is necessary. We shall emphasize the analogy with the case of a Dirichlet form on an L^2 space, which becomes quasi-regular after an enlargement of the space by a zero set.

The structure of invariant subspaces for finite index shifts on the Hardy space

Filippo Bracci
Tor Vergata University, Rome, Italy

Abstract

The famous Beurling theorem provides a concrete characterization of closed invariant subspaces for the shift on the Hardy space H^2 in the unit disc, stating that every such space is of the form fH^2, where f is an inner function. This result can also be interpreted in an operator sense by saying that every closed subspace invariant for the shift is the image of H^2 via an isometry. From this perspective, Beurling's theorem has been extended by Lax, Halmos, and Rovnyak to shifts of any index, proving that a closed subspace is invariant for a shift if and only if it is the image of the space via a quasi-isometry that commutes with the shift (the so-called Beurling-Lax theorem).

In this talk, I will present a generalization of the "concrete" form of Beurling's theorem for the shift on the direct finite sum of H^2. I will show that every closed invariant subspace is given, up to multiplication by an inner function, by the intersection of what we call "determinantal spaces"—which, roughly speaking, are the preimages of shift-invariant subspaces of H^2 by a linear operator constructed through a determinantal operator. The concreteness of such a structure theorem allows us to prove directly, as in the classical Beurling theorem, that the only non-trivial maximal closed shift-invariant subspaces are of codimension one. Using the universality of the (backward) shift in the class of operators with defect less than or equal to the index of the shift, this gives a proof of the following result: every bounded linear operator from a Hilbert space into itself whose defect is finite has a non-trivial closed invariant subspace.

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Q-difference operator extension of Nehari's inequality and applications to majorization results

Teodor Bulboacă
Babeș-Bolyai University, Cluj-Napoca, Romania

Abstract

We gave an extension of the well-known Nehari's inequality [3, p. 168] using the D_q Jackson's *q-derivative operator* and this result will be used to the prove some majorisation problems:

Lemma 1. *If ω is an analytic function in $\mathbb{D}$, such that $|\omega(z)| < 1$, $z \in \mathbb{D}$, then*

$$\left|D_q(\omega(z))\right| \leq \frac{\left|1-\overline{\omega(zq)}\omega(z)\right|}{1-|z|^2 q},\ z \in \mathbb{D},\ (0<q<1).$$

For $q \to 1^-$ in the above result reduces to the Nehari's inequality.

Let $\mathcal{P}$ be the subclass of all analytic functions χ in the open unit disk $\mathbb{D}$, such that χ has positive real part in $\mathbb{D}$ with $\chi(0) = 1$, and let $\mathcal{A}$ denotes the class of functions f analytic in $\mathbb{D}$ usually normalized by $f(0) = f'(0) - 1 = 0$. For a given $\chi \in \mathcal{P}$ and $q \in (0,1)$ we define the family $\mathcal{S}_q(\chi) \subset \mathcal{A}$ by

$$\mathcal{S}_q(\chi) := \left\{ k \in \mathcal{A} : \frac{zD_q k(z)}{k(z)} \prec \chi(z) \right\}.$$

Assuming that k and h are two analytic functions in $\mathbb{D}$, then k is said to be *majorized* by h in $\mathbb{D}$, denoted by $k(z) \ll h(z)$, if there exists an analytic function μ in $\mathbb{D}$ such that $|\mu(z)| \leq 1$ and $k(z) = \mu(z)h(z)$ for all $z \in \mathbb{D}$ (see [1]).

A consequence of the above lemma is the following modified version of majorization problem for the class $\mathcal{S}_q(\chi)$ connected with Theorem 1.1 of [2]:

Theorem 1. *Let l be analytic in $\mathbb{D}$ with $l \not\equiv 0$, and let $h \in \mathcal{S}_q(\chi)$. If $l(z) \ll h(z)$ in $\mathbb{D}$ such that $l \not\equiv c\,h$ with $|c| = 1$, and $q \in (0,1)$, then*

$$|D_q l(z)| \leq |D_q h(z)|,\ |z| \leq r \leq r^*,$$

where r^* is the positive root of the equation

$$(1-\eta)\rho q r^2 + (1+\eta)r - (1-\eta)\rho = 0,$$

with $\eta = \eta(r,q) := \max\limits_{|\zeta|=qr} |\mu(\zeta)|$ and $\rho = \rho(r) := \min\limits_{|z|=r} |\chi(z)|$. The function μ is those that realize the majorization $l(z) \ll h(z)$ in $\mathbb{D}$, shown in the above definition.

The theorem is followed by many particular and special cases obtained for different choice of the parameters and the involved functions.

1. T.H. MacGregor, *Majorization by univalent functions*, Duke Math. J., **34**(1)(1967), 95–102. https://doi.org/10.1215/S0012-7094-67-03411-4

2. N. Hameed Mohammed and E.A. Adegani, *Majorization problems for class of q-starlike functions*, Afr. Mat., **34**(2023), Art. ID 66, 7 pages. https://link.springer.com/article/10.1007/s13370-023-01107-y

3. Z. Nehari, Conformal mapping, MacGraw-Hill Book Company, New York, Toronto and London, 1952.

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Zeros of random holomorphic sections of big line bundles with continuous metrics

Dan Coman
Syracuse University, New York, USA

Abstract

Let X be a compact normal complex space, L be a big holomorphic line bundle on X and h be a continuous Hermitian metric on L. We consider the spaces of holomorphic sections $H^0(X, L^{\otimes p})$ endowed with the inner product induced by $h^{\otimes p}$ and a volume form on X, and prove that the corresponding sequence of normalized Fubini-Study currents converge weakly to the curvature current $c_1(L, h_{\rm eq})$ of the equilibrium metric $h_{\rm eq}$ associated to h. We also show that the normalized currents of integration along the zero divisors of random sequences of holomorphic sections converge almost surely to $c_1(L, h_{\rm eq})$, for very general classes of probability measures on $H^0(X, L^{\otimes p})$. This is joint work with Turgay Bayraktar, George Marinescu, and Viêt-Anh Nguyên.

On boundary behaviour of open, closed mappings of bounded Dirichlet integral

Mihai Cristea
University of Bucharest, Romania

Abstract

We study boundary behaviour of a class of mappings for which an inverse Poletsky modular inequality holds.

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On some coefficients inequalities in one and several variables

Paula Curt
Babeș-Bolyai University, Cluj-Napoca, Romania

Abstract

In this talk we study first the bounds of the difference of moduli of initial successive coefficients, i.e. $||a_{n+1}| - |a_n||$ for $n = 1, 2$ for some subclasses of univalent functions defined on the unit disk of the complex plane (i.e α-convex functions, Janowski starlike and convex functions). Also, we extend the above sharp inequalities to several complex variables. We investigate similar inequalities for α-quasi-convex mappings of type B and Janowski type mappings defined on the open unit ball of a complex Banach space.

The trace operator of quasi-plurisubharmonic functions on compact Kähler manifolds

Tamás Darvas
University of Maryland, USA

Abstract

We introduce the trace operator for quasi-plurisubharmonic functions on compact Kahler manifolds, allowing us to study the singularities of such functions along submanifolds where their generic Lelong numbers vanish. Using this construction we obtain novel Ohsawa-Takegoshi extension theorems and give applications to restricted volumes of big line bundles (joint work with Mingchen Xia).

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Commutativity in Non-elliptic Discrete Iteration

Santiago Diaz-Madrigal
University of Seville, Spain

Abstract

Given a holomorphic self-map of the unit disc $\varphi \in \mathrm{Hol}(\mathbb{D}) \setminus \{d_{\mathbb{D}}\}$, a quite natural and old question is to describe or, at least, to deeply analyse those $\psi \in \mathrm{Hol}(\mathbb{D}) \setminus \{\mathrm{id}_{\mathbb{D}}\}$ which commutes with φ. One of the approaches to this question concerns the behaviour of ψ with respect to a canonical holomorphic model of φ. Namely, and for instance, if φ is non-elliptic and with holomorphic model $(S, h_\varphi, z \mapsto z+1)$, what is the relationship of ψ with this model? In the hyperbolic case, this problem is essentially closed and it holds ψ commutes with φ if and only if ψ has the same Denjoy-Wolff point than φ and there is $c \in \mathbb{R}$ such that $h_\varphi \circ \psi = h_\varphi + c$. However, in the parabolic case, apart from some partial results, the problem is basically open and two questions have been specially considered in the literature:

(1) Assume φ is parabolic of zero hyperbolic step. Is it also true the mimic result for the hyperbolic case in the case?

(2) Assume φ is parabolic of positive hyperbolic step. It is known that the mimic result for the hyperbolic case is false in the case. But, is there any quiet related result which holds in this framework?

In this talk, we provide a positive answer to both questions.

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One family of analytic functions and the Gauß hypergeometric function

Mark Elin
Braude College, Karmiel, Israel

Abstract

The main object of this talk is the two-parameter family of the classes $\mathfrak{A}_s^t,\ s \geq 0, t \in [0,1]$, consisting of functions that are holomorphic in the open unit disk $\mathbb{D}$, normalized by $f(0) = f'(0) - 1 = 0$ and satisfy the inequality

$$\Re\left[(s-1)\frac{f(z)}{z} + f'(z)\right] \geq st,\ z \in \mathbb{D} \setminus \{0\}.$$

This family was investigated from different points of view by several mathematicians. Analytic properties of these classes along their connection with dynamical systems and the semigroup theory are examined.

The most intriguing results appear when we focus on the structure of the whole family $\{\mathfrak{A}_s^t,\ s \geq 0, t \in [0,1]\}$ from the set-theoretic perspective. This insight motivates us to introduce a refined concept of quasi-infima and quasi-suprema, and to establish their complete description.

Unexpectedly, new properties of the Gauß hypergeometric function ${}_2F_1$ play a crucial role in our investigation.

The talk is based on joint works with F. Jacobzon.

Partially Positive Semidefinite Maps on $*$-Semigroupoids

Aurelian Gheondea
Bilkent University, Ankara, Turkey and Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania

Abstract

Motivated by graph C^*-algebras and topolgical groupoids, we consider positive semidefinite maps on $*$-semigroupoids and investigate their dilations to $*$-representations. These maps obey certain degrees of aggregation and, correspondingly, they require certain degrees of orthogonality. There are two types of dilation theorems that we obtain, firstly by unbounded operators and then by bounded operators.

Mappings of finite directional distortion

Anatoly Golberg
Holon Institute of Technology, Israel

Abstract

In our talk, we start with providing the main rigidity inherent of conformal mappings in higher-dimensional Euclidean spaces and presenting some properties of quasiconformal, quasiregular mappings, and mappings of finite distortion. Some background relating to the regularity feature of multidimensional mappings will be also discussed.

Our main tool for studying the above mapping classes rely on directional dilatations and extremal bounds for the conformal moduli. We also involve the multidimensional Teichmüller theorem on separating rings recently established in [1]. It allows us to investigate the boundary correspondence problems for mappings with weaken regularity assumptions, i.e., from Sobolev class $W^{1,n-1}$.

The talk is based on joint works with T. Sugawa and M. Vuorinen.

1. A. Golberg, T. Sugawa, M. Vuorinen, Teichmüller's theorem in higher dimensions and its applications, Comput. Methods Funct. Theory, **20** (2020), pp. 539–558.

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Boundary conformality, hyperbolic distortion and geodesic convexity in the unit disk

Pavel Gumenyuk
Polytechnical University of Milan, Italy

Abstract

The talk is based on two different preprint:

– with I.Efraimidis (Universidad Autónoma de Madrid, Spain) - to appear soon in arXiv;

– and with M.Kourou, A.Moucha, O.Roth (Universität Würzburg, Germany) `https://doi.org/10.48550/arXiv.2410.13965.`

In the first part, we extend a remarkable result proved by A.Solynin in 2007 concerning the geodesic convexity w.r.t. the Poincaré metric in the unit disk of certain level sets, defined in a nutural way for any given holomorphic self-map of the unit disk. The second part is related to the recent works by F.Bracci, D.Kraus, and O.Roth on boundary versions of the Schwarz–Pick and Ahlfors–Schwarz Lemmas. We show that two classical types of conformality of a holomorphic self-map at a boundary point — existence of a finite angular derivative in the sense of Carathéodory and the weaker property of angle preservation — can be characterized via asymptotic proximity of the hyperbolic norm of the derivative to its sharp upper bound (i.e. to 1). An analogous characterization in operator-theoretic terms is given for the stronger of the two boundary conformality properties (i.e. finiteness of Carathéodory's angular derivative). Our results have natural interpretations for the (forward and backward) dynamics of holomorphic self-maps.

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Solutions to the Loewner partial differential equation in infinite dimensions

Hidetaka Hamada
Kyushu Sangyo University, Fukuoka, Japan

Abstract

Let X be a separable reflexive complex Banach space and h be a Herglotz vector field with $Dh(0,t) = A \in L(X,X)$.

In this talk,

(1) We consider the connection of any standard solution and a biholomorphic standard solution to the Loewner PDE.

(2) As a corollary, we obtain the connection between any standard solution and the canonical solution of the Loewner PDE, which gives an improvement of a theorem due to Graham, Hamada, Kohr and Kohr in 2013 (the converse result is also valid).

(3) We obtain a necessary and sufficient condition so that a standard solution of the Loewner PDE can be connected to the canonical solution of the Loewner PDE by a linear mapping, which gives an improvement of a theorem due to Graham, Hamada, Kohr and Kohr in 2013 (the converse result is valid).

(4) As a corollary of the above results, we also obtain a generalization of the results by Becker in 1973 and Duren, Graham, Hamada and Kohr in 2010 to infinite dimensions.

This is a joint work with Gabriela Kohr and Mirela Kohr.

H. Hamada was partially supported by JSPS KAKENHI Grant Number JP22K03363.

Generators of Loewner chains with applications to certain noncommutative stochastic processes on the unit circle

Takahiro Hasebe
Hokkaido University, Sapporo, Japan

Abstract

Noncommutative unitary stochastic processes with monotone independent increments can be characterized by Loewner chains on the unit disk, having Denjoy-Wolff point at 0 and being continuous with respect to time parameter. From a probabilistic point of view, it is natural to look for a suitable notion of "generator" of a Loewner chain. However, because of lack of differentiability about time parameter, the standard method of differential equations is not available. In this talk, we define an integro-differential equation for Loewner chains, define a generator, and compare it with probability theory. Then the convergence of Loewner chains is fully characterized by the convergence of generators, paralleling to the case of additive processes in probability theory. The talk is based on:
T. Hasebe and I. Hotta, Additive processes on the unit circle and Loewner chains, Int. Math. Res. Not. 2022, Issue 22, 17797-17848.

Bohr's phenomena on the unit ball of a complex Banach space

Tatsuhiro Honda
Senshu University, Tokyo, Japan

Abstract

Let H_1 be the Euclidean space $\mathbb{C}^m$ or ℓ_2 and let $\mathbb{B}_{H_1}$ be the unit ball of H_1. In this talk, we will give new generalizations of several results related to the Bohr radius for locally univalent harmonic functions on the unit disc $\mathbb{U}$ in $\mathbb{C}$ to pluriharmonic mappings on $\mathbb{B}_{H_1}$ with values in H_1 which satisfy some assumptions. All of the results are sharp and some of the results give improvements of the results on the unit disc.

This is a joint work with Hidetaka Hamada, and partially supported by JSPS KAKENHI Grant Number 23K03136.

On reachable families of the Loewner differential equation on $\mathbb{B}^n$

Mihai Iancu
Babeș-Bolyai University, Cluj-Napoca, Romania

Abstract

We consider the reachable families of the Loewner differential equation introduced by I. Graham, H. Hamada, G. Kohr, M. Kohr on the Euclidean unit ball $\mathbb{B}^n$ (inspired by the work of O. Roth on the unit disc). We present some results for these families, involving the extreme points of the Carathéodory family on $\mathbb{B}^n$.

An 'inverse Fekete—Szegö problem' and filtration of generators

Fiana Jacobzon
Braude College, Karmiel, Israel

Abstract

In this talk we introduce and explore a question that can be interpreted as an 'inverse Fekete-Szegö problem'. Addressing this problem reveals a connection to the concept of filtration of infinitesimal generators. These filtration classes are inherently significant, especially due to their applications in the study of semigroups of holomorphic mappings within the unit disk. To address the array of questions arising in this context, we define new filtration classes based on a non-linear differential operator:

$$\alpha \cdot \frac{f(z)}{z} + \beta \cdot \frac{zf'(z)}{f(z)} + (1-\alpha-\beta) \cdot \left[1 + \frac{zf''(z)}{f'(z)}\right].$$

We establish key properties of these classes and derive sharp upper bounds for the modulus of the Fekete–Szegö functional over specific filtration classes. Additionally, we present several open problems to inspire further research in this area.

Joint work with Mark Elin (Braude College of Engineering, Karmiel, Israel) and Nikola Tuneski (Ss. Cyril and Methodius University, Skopje, Republic of North Macedonia)

Deformations of functions germs with constant Jacobian ideal

Cezar Joiţa
Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania

Abstract

We consider families of germs of real or complex analytic functions $F_t : (\mathbb{R}^n, 0) \to (\mathbb{R}, 0)$ or $F_t : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ such that the Jacobian ideal (∂F_t) is independent of t. We discuss topological consequences of this independency. Joint work with Matteo Stockinger and Mihai Tibăr.

Normal derivatives on the boundary of Hölder continuous solution of the Poisson equation

Massimo Lanza de Cristoforis
University of Padova, Italy

Abstract

We plan to introduce a distributional form of the normal derivative on the boundary for Hölder continuous solutions of the Poisson equation, which may have infinite Dirichlet integral. Then we characterize the space for the first order traces of the normal derivatives and we present some application to the solution of the Neumann problem with distributional data. We exploit the layer potential theoretic method.

On Serrin-Type Spatially-Periodic Solutions for Evolution Anisotropic Navier-Stokes Equations

Sergey E. Mikhailov
Brunel University London, UK

Abstract

We consider evolution (non-stationary) space-periodic solutions to the n-dimensional non-linear Navier-Stokes equations of anisotropic fluids with the viscosity coefficient tensor variable in space and time and satisfying the relaxed ellipticity condition. Employing the Galerkin algorithm, we prove the existence of Serrin-type solutions, that is, the weak solutions with the velocity in the periodic space $L_2(0,T;\dot{\mathbf{H}}^{n/2}_{\#\sigma})$, $n \geq 2$. The solution uniqueness and regularity results are also discussed. The talk is based on the following papers.

1. S.E. Mikhailov, Spatially-Periodic Solutions for Evolution Anisotropic Variable-Coefficient Navier-Stokes Equations: I. Weak Solution Existence. *Mathematics*, **12**(12), 1817 (2024), 1–27, https://doi.org/10.3390/math12121817.

2. S.E. Mikhailov, Spatially-Periodic Solutions for Evolution Anisotropic Variable-Coefficient Navier-Stokes Equations: II. Serrin-Type Solutions. *arXiv2407.05488*, 2024, 44p.

Scattering theory and integral representation formulas

Dorina Mitrea
Baylor University, Texas, USA

Abstract

In this talk I will answer the following basic question:

What are the optimal assumptions, of geometric and analytic nature, which guarantee that a null-solution u of the Helmholtz operator $\Delta + k^2$ in an exterior domain Ω can be represented in terms of layer potentials naturally associated with the said Helmholtz operator and given domain?

This work, at the interface between Geometric Measure Theory, Harmonic Analysis, Scattering Theory, and Clifford Analysis, generalizes and unifies classical results of Sommerfeld, Weyl, Müller, and Calderón.

Boundary Value Problems for Higher Order Differential Operators in Vanishing Chord Arc Domains

Irina Mitrea
Temple University, Philadelphia, USA

Abstract

One of the most effective methods for solving boundary value problems for basic equations of mathematical physics in a domain is the method of layer potentials. Its essence is to reduce the entire problem to an integral equation on the boundary of the domain which is then solved using Fredholm theory.

Until recently, this approach has been primarily used in connection with second order operators for which a sophisticated and far reaching theory exists. This stands in contrast with the case of higher order operators (arising for instance in plate elasticity) for which the theory is significantly less developed. In this talk I will discuss recent results aimed at extending the method of singular integral operators (of layer potential type) and the Fredholm theory approach to the higher order case, in vanishing chord arc domains.

The Extension Problem for CR functions on Uniformly Rectifiable Sets

Marius Mitrea
Baylor University, Texas, USA

Abstract

In this talk I will be concerned with extension phenomena in several complex variables. Virtually all results of this nature originate in the pioneering work of Friedrich Hartogs who used Cauchy's integral formula to prove in 1906 his famous theorem to the effect that if $n \geq 2$ and Ω is an open bounded subset of $\mathbb{C}^n$ with connected boundary, then any holomorphic function f in a neighborhood U of $\partial\Omega$ extends to a holomorphic function in Ω. In the limiting case when the neighborhood U shrinks to $\partial\Omega$, the standard holomorphicity assumption for f should be replaced by the demand that f is a CR-function on $\partial\Omega$. The latter condition is meaningful under suitable assumptions on f and $\partial\Omega$, a scenario in which we shall refer to this question as the Extension Problem (EP). The history of EP is very distinguished, but progress has stopped at domains of class C^1. The goal here is to solve EP merely assuming that the underlying domain has a uniformly rectifiable boundary.

Continuity of certain capacity-like quantities related to chordal Loewner chains

Takuya Murayama
Kyushu University, Fukuoka, Japan

Abstract

In several applications of Loewner chains, it is necessary to consider a class of conformal mappings with a common boundary fixed point. If we take the upper half-plane as our canonical space and the point at infinity as the fixed point, then a fundamental role is played by the angular residue at infinity of those conformal mappings. This residue is also called the half-plane capacity (of the corresponding boundary hulls). In this talk, on the basis of two of my (joint) works, I shall present some results on the continuity of this capacity-like quantity when the images of the conformal mappings vary continuously in the sense of Carathéodory. If time permits, I shall also discuss 1) what meaning these results have in a certain application of "chordal" Loewner chains; 2) how these results are established for conformal mappings on finitely connected domains.

Mixed norm estimates via the Helicoidal Method

Camil Muscalu
Cornell University, New York, USA

Abstract

The goal of the talk is to explain how the Helicoidal Method that we developed in the last few years can also offer a new paradigm for proving mixed norm estimates for operators in Harmonic Analysis. Most likely, we will concentrate on the (least technical) case of paraproducts, and describe a natural, complete, mixed norm estimates extension, of the classical Coifman-Meyer theorem. Joint work with Cristina Benea.

Elliptic problems with 'log-normal' random coefficients

Victor Nistor
Université de Lorraine, Metz, France

Abstract

We consider a family of elliptic problems whose coefficients define random variables in a certain probability space. We prove uniform estimates for the solutions that are polynomial in the norms of the coefficients. In particular, if the coefficients are 'log-normal' in a suitable sense, we show that (the norm of) the solutions define an integrable random variable. (My talk is based on joint works with C. Bacuta, M. Kohr, S. Labrunie, Hengguang Li, and H. Mohsen).

A lot of one kind, or a little bit of different kinds, which is better?

Mihai Pascu
Transilvania University of Braşov, Romania

Abstract

If we were to ask this question to a nutritionist, the answer would surely be that the second option is better.

In the present talk we will try to explore the advantages and disadvantages of the two options, in the context of the representation of analytic function. A bit more precise, we will try to compare the representation of an analytic function by means of its Taylor series development at a point (that is, knowing a lot of information about the function at a single point, i.e. the value of the function and all its derivatives at a point) and the representation of the function given by the values of the function at a sequence of points which accumulate to the given point (a little bit of information about different points).

The size of critical sets

Cornel Pintea
Babeș-Bolyai University, Cluj-Napoca, Romania

Abstract

We evaluate the minimal size of critical sets of functions between differential manifolds, both by cardinality and topological dimension. The topological dimension is used for pairs of manifolds with the property that each function between them has infinitely many critical points.

Faber-Tietz forms and series on Riemann surfaces

Eric Schippers
University of Manitoba, Canada

Abstract

Faber polynomials are polynomials associated to a conformal map onto a planar domain. It is classical that one can approximate holomorphic functions on the complement of this domain by series of Faber polynomials, and there is a well-developed theory of Faber series in various analytic settings. Results on approximation in the Bergman space norm by Faber series are surprisingly recent and hold precisely for quasicircles. Based on work of H. Tietz, we generalize Faber polynomials and Faber series to compact Riemann surfaces with conformal maps onto quasidisks, and show that one-forms on the complement have a convergent Faber-Tietz series in L^2. Joint work with M. Shirazi.

Two dimensional tomography via the A-analytic maps on the space of sequences

Alexandru Tămăşan
University of Central Florida, USA

Abstract

The notion of A-analyticity was introduced by A. Bukhgeim in the mid 1990s in order to solve an outstanding open problem in planar tomography. I will present some of Bukhgeim's original ideas, and how they extend naturally to a system of Beltrami-like equations for A-analytic maps. I will briefly describe their practical use in tensor tomography.

Pseudodifferential Operators with Symbols of Potential Type

Wolfgang L. Wendland
University of Stuttgart, Institute for Applied Analysis and Numerical Simulation & SIMTECH, Germany

Abstract

Chapter 1: Pseudohomogeous Schwartz kernels and pseudodifferential operators $\Psi hk_\kappa(\Omega)$.
Chapter 2: Invariance properties od Schwartz kernels $\Psi hk_\kappa(\Omega)$.
Chapter 3: Pseudodifferential operators of rational type.
Chapter 4: The Stokes equations.

The lecture is based on the book by G.C. Hsiao and the Speaaker:Boundary Integral Equations. Second Edition 2021, Springer–Verlag, Heidelberg.

Contributed Talks

An extension operator of Roper-Suffridge and Graham-Kohr type

Mihai Aron
Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

In this paper, we generalize the Roper-Suffridge-Graham-Kohr extension operator, $f \mapsto \Psi_{n,\alpha,\beta}$, where f is normalized locally univalent function on the unit disc of the complex plane and

$$\Psi_{n,\alpha,\beta}(f)(z) = \left(f(z_1), \left[\frac{f(z_1)}{z_1} \right]^{\alpha} [f'(z_1)]^{\beta} \tilde{z}, \right),$$

where $z = (z_1, \tilde{z})$ belongs to the unit ball of $\mathbb{C}^n$.

We address to the problem of preserving starlikeness and convexity through the new operator. The newly introduced operator provides a new method for constructing starlike functions on the unit ball of $\mathbb{C}^n$. Also, we improve the result regarding preservation of starlikness in case of extension operator $\Psi_{n,\alpha,\beta}$. Finally, we propose an open problem and present a brief introductory study on it.

The Graham-Kohr extension operator in complex Banach spaces

Eduard Grigoriciuc
Babeș-Bolyai University & ICTP, Cluj-Napoca, Romania

Abstract

Inspired by the recent results obtained by I. Graham, H. Hamada, G. Kohr and M. Kohr, we study some important properties of the Graham-Kohr extension operator (introduced by I. Graham and G. Kohr in 2002) in complex Banach spaces.