

Boundary conformality, hyperbolic distortion and geodesic convexity in the unit disk

Pavel Gumenyuk
Polytechnical University of Milan, Italy

Abstract

The talk is based on two different preprint:

- with I.Efraimidis (Universidad Autónoma de Madrid, Spain) - to appear soon in arXiv;
- and with M.Kourou, A.Moucha, O.Roth (Universität Würzburg, Germany)
<https://doi.org/10.48550/arXiv.2410.13965>.

In the first part, we extend a remarkable result proved by A.Solynin in 2007 concerning the geodesic convexity w.r.t. the Poincaré metric in the unit disk of certain level sets, defined in a natural way for any given holomorphic self-map of the unit disk. The second part is related to the recent works by F.Bracci, D.Kraus, and O.Roth on boundary versions of the Schwarz–Pick and Ahlfors–Schwarz Lemmas. We show that two classical types of conformality of a holomorphic self-map at a boundary point — existence of a finite angular derivative in the sense of Carathéodory and the weaker property of angle preservation — can be characterized via asymptotic proximity of the hyperbolic norm of the derivative to its sharp upper bound (i.e. to 1). An analogous characterization in operator-theoretic terms is given for the stronger of the two boundary conformality properties (i.e. finiteness of Carathéodory’s angular derivative). Our results have natural interpretations for the (forward and backward) dynamics of holomorphic self-maps.