

One family of analytic functions and the Gauß hypergeometric function

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Abstract

The main object of this talk is the two-parameter family of the classes \mathfrak{A}_s^t , $s \geq 0, t \in [0, 1]$, consisting of functions that are holomorphic in the open unit disk \mathbb{D} , normalized by $f(0) = f'(0) - 1 = 0$ and satisfy the inequality

$$\Re \left[(s-1) \frac{f(z)}{z} + f'(z) \right] \geq st, \quad z \in \mathbb{D} \setminus \{0\}.$$

This family was investigated from different points of view by several mathematicians. Analytic properties of these classes along their connection with dynamical systems and the semigroup theory are examined.

The most intriguing results appear when we focus on the structure of the whole family $\{\mathfrak{A}_s^t, s \geq 0, t \in [0, 1]\}$ from the set-theoretic perspective. This insight motivates us to introduce a refined concept of quasi-infima and quasi-suprema, and to establish their complete description.

Unexpectedly, new properties of the Gauß hypergeometric function ${}_2F_1$ play a crucial role in our investigation.

The talk is based on joint works with F. Jacobzon.