Workshop dedicated to the memory of Professor Gabriela Kohr
(4th edition)
Geometric Function Theory in Several Complex Variables and Complex Banach Spaces

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Commutativity in Non-elliptic Discrete Iteration

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Abstract

Given a holomorphic self-map of the unit disc $\varphi \in \mathrm{Hol}(\mathbb{D}) \setminus \{d_{\mathbb{D}}\}$, a quite natural and old question is to describe or, at least, to deeply analyse those $\psi \in \mathrm{Hol}(\mathbb{D}) \setminus \{\mathrm{id}_{\mathbb{D}}\}$ which commutes with φ. One of the approaches to this question concerns the behaviour of ψ with respect to a canonical holomorphic model of φ. Namely, and for instance, if φ is non-elliptic and with holomorphic model $(S, h_\varphi, z \mapsto z+1)$, what is the relationship of ψ with this model? In the hyperbolic case, this problem is essentially closed and it holds ψ commutes with φ if and only if ψ has the same Denjoy-Wolff point than φ and there is $c \in \mathbb{R}$ such that $h_\varphi \circ \psi = h_\varphi + c$. However, in the parabolic case, apart from some partial results, the problem is basically open and two questions have been specially considered in the literature:

(1) Assume φ is parabolic of zero hyperbolic step. Is it also true the mimic result for the hyperbolic case in the case?

(2) Assume φ is parabolic of positive hyperbolic step. It is known that the mimic result for the hyperbolic case is false in the case. But, is there any quiet related result which holds in this framework?

In this talk, we provide a positive answer to both questions.