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The structure of invariant subspaces for finite index shifts on the Hardy space

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Abstract

The famous Beurling theorem provides a concrete characterization of closed invariant subspaces for the shift on the Hardy space H^2 in the unit disc, stating that every such space is of the form fH^2, where f is an inner function. This result can also be interpreted in an operator sense by saying that every closed subspace invariant for the shift is the image of H^2 via an isometry. From this perspective, Beurling's theorem has been extended by Lax, Halmos, and Rovnyak to shifts of any index, proving that a closed subspace is invariant for a shift if and only if it is the image of the space via a quasi-isometry that commutes with the shift (the so-called Beurling-Lax theorem).

In this talk, I will present a generalization of the "concrete" form of Beurling's theorem for the shift on the direct finite sum of H^2. I will show that every closed invariant subspace is given, up to multiplication by an inner function, by the intersection of what we call "determinantal spaces" – which, roughly speaking, are the preimages of shift-invariant subspaces of H^2 by a linear operator constructed through a determinantal operator. The concreteness of such a structure theorem allows us to prove directly, as in the classical Beurling theorem, that the only non-trivial maximal closed shift-invariant subspaces are of codimension one. Using the universality of the (backward) shift in the class of operators with defect less than or equal to the index of the shift, this gives a proof of the following result: every bounded linear operator from a Hilbert space into itself whose defect is finite has a non-trivial closed invariant subspace.