

Sign Test

Problem 1 The following data give temperature (in $^{\circ}\text{C}$) for 20 days: 22 18 17 16 13 20 19 21 20 16 14 17 21 21 17 17 17 22 22 21. Test if the median is $m = 19$.

Problem 2 The number of defective electrical fuses produced by each of two production lines, A and B, was recorded daily for a period of 10 days, with the results shown in Table 1. Assume that both production lines produced the same daily output. Compare the number of defectives produced by A and B each day and let M equal the number of days when A exceeded B. Do the data present sufficient evidence to indicate that either production line produces more defectives than the other? State the null hypothesis to be tested and use M as a test statistic.

Day	A	B
1	172	201
2	165	179
3	206	159
4	184	192
5	174	177
6	142	170
7	190	182
8	169	179
9	161	169
10	200	210

Table 1: Data for Problem 2

Problem 3 (Drugs) Clinical data concerning the effectiveness of two drugs for treating a disease were collected from ten hospitals. The number of patients treated with the drugs differed for the various hospitals. The data are given in the table 2.

- Do the data indicate a difference in the recovery rates for the two drugs? Give the associated p -value.
- Why might it be inappropriate to use the t test to analyze the data?

Problem 4 The data in the file `chickenembrios.txt` are a subset of the data obtained by Oppenheim (1968) in an experiment investigating light responsivity in chick embryos. The subjects were white leghorn chick embryos, and the behavioral response measured in the investigation was beak-clapping (i.e., the rapid opening and closing of the beak that occurs during the latter one-third of incubation in chick embryos). (Gottlieb (1965) had previously shown that changes

Hospital	Drug A			Drug B		
	Number Treated	Number Recovered	Percentage Recovered	Number Treated	Number Recovered	Percentage Recovered
1	84	63	75.0	96	82	85.4
2	63	44	69.8	83	69	83.1
3	56	48	85.7	91	73	80.2
4	77	57	74.0	47	35	74.5
5	29	20	69.0	60	42	70.0
6	48	40	83.3	27	22	81.5
7	61	42	68.9	69	52	75.4
8	45	35	77.8	72	57	79.2
9	79	57	72.2	89	76	85.4
10	62	48	77.4	46	37	80.4

Table 2: Recovery rates for two drugs

in the rate of beak-clapping constituted a sensitive indicator of auditory responsiveness in chick embryos.) The embryos were placed in a dark chamber 30 min before the initiation of testing. Then ten 1-min readings were taken in the dark, and at the end of this 10-min period, a single reading was obtained for a 1-min period of illumination. File `chickenembrios.txt` gives the average number of claps per minute during the dark period (XD) and the corresponding rate during the period of illumination (YL) for 25 chick embryos.

The Wilcoxon Signed-Rank Test for a Matched-Pairs Experiment

Problem 5 Due to oven-to-oven variation, a matched-pairs experiment was used to test for differences in cakes prepared using mix A and mix B. Two cakes, one prepared using each mix, were baked in each of six different ovens (a total of 12 cakes). Test the hypothesis that there is no difference in population distributions of cake densities using the two mixes. What can be said about the attained significance level? The original data and differences in densities (in ounces per cubic inch) for the six pairs of cakes are shown in Table 3.

Problem 6 The accompanying Table 4 gives the scores of a group of 15 students in mathematics and art.

- (a) Use Wilcoxon's signed-rank test to determine if the locations of the distributions of scores for these students differ significantly for the two subjects. Give bounds for the p-value and indicate the appropriate conclusion with $\alpha = .05$.

A	B	Difference $A - B$	Absolute Difference	Rank of Absolute Difference
.135	.129	.006	.006	3
.102	.120	-.018	.018	5
.108	.112	-.004	.004	1.5
.141	.152	-.011	.011	4
.131	.135	-.004	.004	1.5
.144	.163	-.019	.019	6

Table 3: Paired data and their differences for Problem 5

Student	Math	Art	Student	Math	Art
1	22	53	9	62	55
2	37	68	10	65	74
3	36	42	11	66	68
4	38	49	12	56	64
5	42	51	13	66	67
6	58	65	14	67	73
7	58	51	15	62	65
8	60	71			

Table 4: Students scores for Math and Art, Problem 6

(b) State specific null and alternative hypotheses for the test that you conducted in part (a).

Independent Random Samples: Wilcoxon and Mann-Witney

Problem 7 The bacteria counts per unit volume are shown in Table 5 for two types of cultures, I and II. Four observations were made for each culture. Let n_1 and n_2 represent the number of observations in samples I and II, respectively. For the data given in Table 5, the corresponding ranks are as shown in Table 6. Do these data present sufficient evidence to indicate a difference in the locations of the population distributions for cultures I and II?

I	II
27	32
31	29
26	35
25	28

Table 5: Data for Problem 7

	I	II
	3	7
	6	5
	2	8
	1	4
Rank sum:	12	24

Table 6: Ranks for Example 7

Problem 8 *Two plastics, each produced by a different process, were tested for ultimate strength. The measurements in the accompanying table represent breaking loads in units of 1000 pounds per square inch. Do the data present evidence of a difference between the locations of the distributions of ultimate strengths for the two plastics? Test by using the Mann–Whitney U test with a level of significance as near as possible to $\alpha = .10$.*

Plastic 1	Plastic 2
15.3	21.2
18.7	22.4
22.3	18.3
17.6	19.3
19.1	17.1
14.8	27.7

Table 7: Plastic strength, Problem 8

The Kruskal–Wallis Test for the One-Way Layout

Problem 9 *A quality control engineer has selected independent samples from the output of three assembly lines in an electronics plant. For each line, the output of ten randomly selected hours of production was examined for defects. Do the data in Table 8 provide evidence that the probability distributions of the number of defects per hour of output differ in location for at least two of the lines? Use $\alpha = .05$. Also give the p -value associated with the test.*

Problem 10 *The table 9 contains data on the leaf length for plants of the same species at each of four swampy underdeveloped sites. At each site, six plants were randomly selected. For each plant, ten leaves were randomly selected, and the mean of the ten measurements (in centimeters) was recorded for each plant from each site. Use the Kruskal–Wallis H test to determine whether there is sufficient evidence to claim that the distribution of mean leaf lengths differ in location for at least two of the sites. Use $\alpha = .05$. Bound or find the approximate p -value.*

Line 1		Line 2		Line 3	
Def	R	Def	R	Def	R
6	5	34	25	13	9.5
38	27	28	19	35	26
3	2	42	30	19	15
17	13	13	9.5	4	3
11	8	40	29	29	20
30	21	31	22	0	1
15	11	9	7	7	6
16	12	32	23	33	24
25	17	39	28	18	14
5	4	27	18	24	16
$R_1 = 120$		$R_2 = 210.5$		$R_3 = 134.5$	

Table 8: Data for Problem 9

Site	MeanLeafLength (cm)					
1	5.7	6.3	6.1	6.0	5.8	6.2
2	6.2	5.3	5.7	6.0	5.2	5.5
3	5.4	5.0	6.0	5.6	4.0	5.2
4	3.7	3.2	3.9	4.0	3.5	3.6

Table 9: Data for Problem 10

The Friedman Test for Randomized Block Designs

Problem 11 *An experiment to compare completion times for three technical tasks was performed in the following manner. Because completion times may vary considerably from person to person, each of the six technicians was asked to perform all three tasks. The tasks were presented to each technician in a random order with suitable time lags between the tasks. Do the data in Table 10 present sufficient evidence to indicate that the distributions of completion times for the three tasks differ in location? Use $\alpha = .05$. Give bounds for the associated p -value.*

Techn	Task A	Rank	Task B	Rank	Task C	Rank
1	1.21	1	1.56	3	1.48	2
2	1.63	1.5	2.01	3	1.63	1.5
3	1.42	1	1.70	2	2.06	3
4	1.16	1	1.27	2.5	1.27	2.5
5	2.43	2	2.64	3	1.98	1
6	1.94	1	2.81	3	2.44	2
Rank sum		7.5		16.5		12

Table 10: Completion times for three tasks

Ingot	Sealant		
	I	II	III
1	4.6	4.2	4.9
2	7.2	6.4	7.0
3	3.4	3.5	3.4
4	6.2	5.3	5.9
5	8.4	6.8	7.8
6	5.6	4.8	5.7
7	3.7	3.7	4.1
8	6.1	6.2	6.4
9	4.9	4.1	4.2
10	5.2	5.0	5.1

Table 11: Corrosion data example 12

Problem 12 *Corrosion of metals is a problem in many mechanical devices. Three sealants used to help retard the corrosion of metals were tested to see whether there were any differences among them. Samples from ten different ingots of the same metal composition were treated with each of the three sealants, and the amount of corrosion was measured after exposure to the same environmental conditions for 1 month. The data are given in the Table 11. Is there any evidence of a difference in the abilities of the sealants to prevent corrosion? Test using $\alpha = .05$.*

The Runs Test

Problem 13 *A true-false examination was constructed with the answers running in the following sequence:*

T F F T F T F T T F T F F T F T F T T F.

Does this sequence indicate a departure from randomness in the arrangement of T and F answers?

Problem 14 *Paper is produced in a continuous process. Suppose that a brightness measurement Y is made on the paper once every hour and that the results appear as shown in Figure 1. The average y for the 15 sample measurements appears as shown. Notice the deviations about y . Do these data indicate a lack of randomness and thereby suggest periodicity and lack of control in the process?*

Problem 15 *A union supervisor claims that applicants for jobs are selected without regard to race. The hiring records of the local—one that contains all male members—gave the following sequence of White (W) and Black (B) hirings:*

W W W W B W W W B B W B B

Do these data suggest a nonrandom racial selection in the hiring of the union's members?

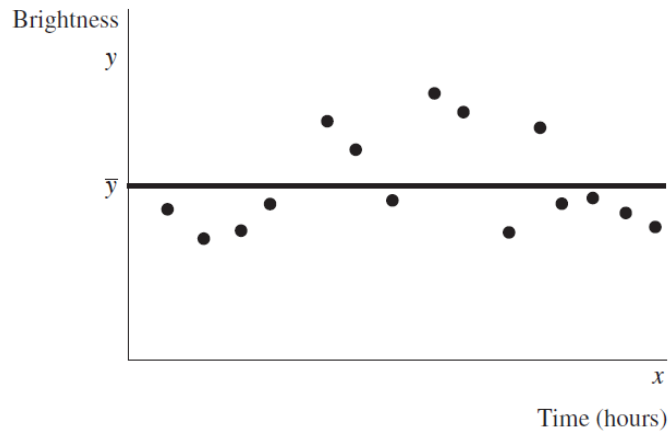


Figure 1: Paper brightness versus time

Teacher	Judge's Rank	Examination Score
1	7	44(1)
2	4	72(5)
3	2	69(3)
4	6	70(4)
5	1	93(8)
6	3	82(7)
7	8	67(2)
8	5	80(6)

Table 12: Data for science teachers

Rank Correlation Coefficient

Problem 16 Suppose that eight elementary-science teachers have been ranked by a judge according to their teaching ability, and all have taken a national teachers' examination. The data are given in Table 12. Do the data suggest agreement between the judge's ranking and the examination score? Alternatively, we might express this question by asking whether a correlation exists between the judge's ranking and the ranks of examination scores. Calculate r_S for the judge's ranking and examination score data from Table 12. Test the hypothesis of no association between populations.

Problem 17 (Proline and Collagen in Liver Cirrhosis) Kershenobich, Fierro, and Rojkind (1970) have studied the relation between the free pool of proline and collagen content in human liver cirrhosis. The data below (Table 13) are based on an analysis of cirrhotic livers from seven patients, each having a histological

Patient	Xi Proline	Yi Collagen
1	7.1	2.8
2	7.1	2.9
3	7.2	2.8
4	8.3	2.6
5	9.4	3.5
6	10.5	4.6
7	11.4	5.0

Table 13: Data for Problem 17

diagnosis of portal cirrhosis. We are interested in assessing whether there is a positive relationship between the total collagen and the free proline in cirrhotic livers.