Problem 1 *At temperatures approaching absolute zero ($-273°C$), helium exhibits traits that defy many laws of conventional physics. An experiment has been conducted with helium in solid form at various temperatures near absolute zero. The solid helium is placed in a dilution refrigerator along with a solid impure substance, and the fraction (in weight) of the impurity passing through the solid helium is recorded. (The phenomenon of solids passing directly through solids is known as quantum tunneling.) The data are given in the following table.*

$°C$ *Temperature* (x)	*Proportion of Impurity Passing Through Helium* (y)
-262.0	.315
-265.0	.202
-256.0	.204
-267.0	.620
-270.0	.715
-272.0	.935
-272.4	.957
-272.7	.906
-272.8	.985
-272.9	.987

(a) *Fit a least-squares line to the data.*

(b) *Test the null hypothesis $H_0 : \beta_1 = 0$ against the alternative hypothesis $H_a : \beta_1 < 0$, at the $\alpha = .01$ level of significance.*

(c) *Find a 95% prediction interval for the percentage of the solid impurity passing through solid helium at $-273°C$. (This value of x is outside the experimental region where use of the model for prediction may be dangerous.)*

Problem 2 *A study was conducted to determine whether a linear relationship exists between the breaking strength y of wooden beams and the specific gravity x of the wood. Ten randomly selected beams of the same cross-sectional dimensions were stressed until they broke. The breaking strengths and the density of the wood are shown in the accompanying table for each of the ten beams.*

Beam Specific Gravity (x)	*Strength* (y)
1.499	11.14
2.558	12.74
3.604	13.13
4.441	11.51
5.550	12.38
6.528	12.60
7.418	11.13
8.480	11.70
9.406	11.02
10.467	11.41

(a) *Fit the model* $Y = \beta_0 + \beta_1 x + \varepsilon$.

(b) *Test* $H_0 : \beta 1 = 0$ *against the alternative hypothesis,* $Ha : \beta_1 \neq 0$.

(c) *Estimate the mean strength for beams with specific gravity .590, using a 90% confidence interval.*

Problem 3 *A response* Y *is a function of three independent variables* x_1, x_2, *and* x_3 *that are related as follows:*

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon.$$

(a) *Fit this model to the* $n = 7$ *data points shown in the accompanying table.*

y	x_1	x_2	x_3
1	-3	5	-1
0	-2	0	1
0	-1	-3	1
1	0	-4	0
2	1	-3	-1
3	2	0	-1
3	3	5	1

(b) *Predict* Y *when* $x_1 = 1$, $x_2 = -3$, $x_3 = -1$. *Compare with the observed response in the original data. Why are these two not equal?*

(c) *Do the data present sufficient evidence to indicate that* x_3 *contributes information for the prediction of* Y *? (Test the hypothesis* $H_0 : \beta_3 = 0$, *using* $\alpha = .05$.*)*

(d) *Find a 95% confidence interval for the expected value of Y, given* $x_1 = 1$, $x_2 = -3$, *and* $x_3 = -1$.

(e) *Find a 95% prediction interval for* Y, *given* $x_1 = 1$, $x_2 = -3$, *and* $x_3 = -1$.

Problem 4 *Show that the least-squares prediction equation* $\widehat{y} = \widehat{\beta}_0 + \widehat{\beta}_1 x_1 + \cdots + \widehat{\beta}_k x_k$ *passes through the point* $(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k, \overline{y})$.

Problem 5 *An experiment was conducted to determine the effect of pressure and temperature on the yield of a chemical. Two levels of pressure (in pounds per square inch, psi) and three of temperature were used:*

Pressure (psi)	*Temperature (°F)*
50	100
80	200
	300

One run of the experiment at each temperature–pressure combination gave the data listed in the following table.

Yield	*Pressure (psi)*	*Temperature (°F)*
21	50	100
23	50	200
26	50	300
22	80	100
23	80	200
28	80	300

(a) *Fit the model* $Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_2^2 + \varepsilon$*, where* $x_1 =$ *pressure and* x_2 *=temperature.*

(b) *Test to see whether* β_3 *differs significantly from zero, with* $\alpha = .05$*.*

(c) *Test the hypothesis that temperature does not affect the yield, with* $\alpha = .05$*.*

Problem 6 *The data in the accompanying table come from the comparison of the growth rates for bacteria types A and B. The growth Y recorded at five equally spaced (and coded) points of time is shown in the table.*

	Time				
Bacteria type	-2	-1	0	1	2
A	8.0	9.0	9.1	10.2	10.4
B	10.0	10.3	12.2	12.6	13.9

(a) *Fit the linear model*

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \varepsilon$$

to the $n = 10$ *data points. Let* $x_1 = 1$ *if the point refers to bacteria type B and let* $x_1 = 0$ *if the point refers to type A. Let* $x_2 =$ *coded time.*

(b) *Plot the data points and graph the two growth lines. Notice that* β_3 *is the difference between the slopes of the two lines and represents time–bacteria interaction.*

(c) *Predict the growth of type A at time* $x_2 = 0$ *and compare the answer with the graph. Repeat the process for type B.*

(d) *Do the data present sufficient evidence to indicate a difference in the rates of growth for the two types of bacteria?*

(e) *Find a 90% confidence interval for the expected growth for type B at time* $x_2 = 1$*.*

(f) *Find a 90% prediction interval for the growth Y of type B at time* $x_2 = 1$*.*