

## V. Şiruri. Probleme propuse.

I. Calculați limitele:

1.  $\lim_{n \rightarrow \infty} a_n, a_n = \sum_{k=1}^n \frac{\cos k}{n^2 + kn + k^2}$       2.  $\lim_{n \rightarrow \infty} a_n, a_n = \frac{1^p + 2^p + \dots + n^p}{n^{p+1}}, p \in \mathbb{N}$

3.  $\lim_{n \rightarrow \infty} \frac{\ln n}{n}$

4.  $\lim_{n \rightarrow \infty} \sqrt[n]{n}$

5.  $\lim_{n \rightarrow \infty} \frac{n}{\sqrt[n]{n!}}$

6.  $\lim_{n \rightarrow \infty} \frac{n^n}{n!}$

7.  $\lim_{n \rightarrow \infty} a_n, a_n = \frac{2^3-1}{2^3+1} \cdot \frac{3^3-1}{3^3+1} \cdot \frac{4^3-1}{4^3+1} \cdot \dots \cdot \frac{n^3-1}{n^3+1}$

8.  $\lim_{n \rightarrow \infty} a_n, a_n = \frac{1}{3^2} + \frac{2}{15^2} + \frac{3}{35^2} + \dots + \frac{n}{[(2n-1)(2n+1)]^2}$

9.  $\lim_{n \rightarrow \infty} x_n, x_{n+1} = \sqrt{\alpha + x_n}, x_0 = \sqrt{\alpha}, \alpha > 0$

ii. Dacă  $\lim_{n \rightarrow \infty} a_n = a \in \mathbb{R}^*$  calculați:  $\lim_{n \rightarrow \infty} b_n, \lim_{n \rightarrow \infty} c_n$  și limite din sticlă

$$b_n = \frac{a_1 + \dots + a_n}{n}, \quad c_n = \frac{n}{\frac{1}{a_1} + \dots + \frac{1}{a_n}}, \quad d_n = \sqrt[n]{a_1 \cdot a_2 \cdot \dots \cdot a_n}$$

iii. Calculați:

1.  $\lim_{n \rightarrow \infty} \frac{2^n \cdot n!}{n^n}$

2.  $\lim_{n \rightarrow \infty} \frac{2^n}{n!}$

3.  $\lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^2+3}{3n+1}}$

4.  $\lim_{n \rightarrow \infty} x_n, x_{n+1} = \frac{1}{2} \left( x_n + \frac{1}{x_n} \right), x_1 = a, a > 1$

iv. Să se demonstreze că şirul  $a_n$  este convergent

1.  $a_n = 1 + \frac{1}{2^2} + \dots + \frac{1}{n^2}$

2.  $a_n = \frac{\cos 1!}{1 \cdot 2} + \frac{\cos 2!}{2 \cdot 3} + \dots + \frac{\cos n!}{n(n+1)}$

v. Calculați:

1.  $\lim_{n \rightarrow \infty} \sqrt[n]{n!}$

2.  $\lim_{n \rightarrow \infty} \sqrt[n]{\sin \frac{\pi}{2} \cdot \sin \frac{\pi}{3} \cdot \dots \cdot \sin \frac{\pi}{n}}$

3.  $\lim_{n \rightarrow \infty} \sqrt[n]{n! \cdot \sin \frac{\pi}{2} \cdot \sin \frac{\pi}{3} \cdot \dots \cdot \sin \frac{\pi}{n}}$