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NON-LINEAR DENSITY VARIATION EFFECTS ON THE FULLY DEVELOPED MIXED CONVECTION FLOW IN A VERTICAL CHANNEL

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1. Introduction

Heat transfer in channels occurs in many industrial processes and natural phenomena. It has been, therefore, the subject of many detailed, mostly numerical studies for different flow configurations. Most of the interest in this subject is due to its practical applications, for example, in the design of cooling systems for electronic devices and in the field of solar energy collection. Some of the published papers, such as by Aung [1], Aung et al. [2], Aung and Worku [3, 4], Barletta [5, 6], and Boulama and Galanis [7], are concerned with the evaluation of the temperature and velocity profiles for the vertical parallel-flow fully developed regime. As is well known, heat exchangers technology involves convective flows in vertical channels. In most cases, these flows imply conditions of uniform heating of a channel, which can be modelled either by uniform wall temperature (UWT) or uniform wall heat flux (UHF) thermal boundary conditions.

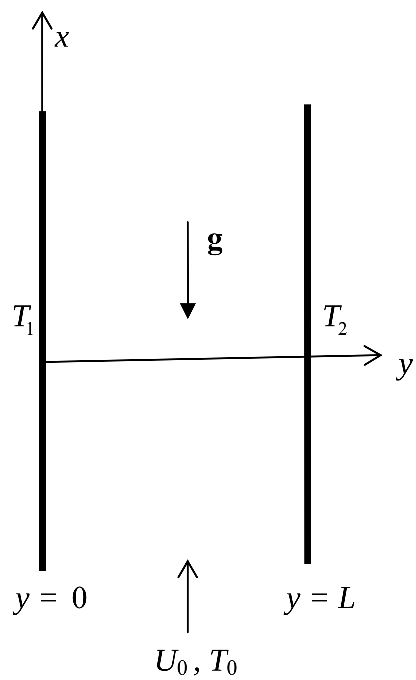

Fig. 1. *Geometry of the problem and co-ordinate system*

In the present paper, the effect of the non-linear density variation on the steady mixed convection flow in a long vertical channel is investigated. It is assumed that the density variation is a combination of the linear and quadratic terms of temperature, see Vajravelu and Sastri [8], that is

$$\Delta\rho/\rho = -\beta(T - T_0) - \gamma(T - T_0)^2 \qquad (1)$$

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where β and γ are coefficients of thermal expansion, ρ is the density and T_0 is the reference temperature. The first term in this equation is the classical Boussinesq approximation, while the second term corresponds to water at 4°C. Under this assumption the governing equations are expressed in non-dimensional form and are solved both analytically and numerically.

2. Basic equations

Consider a viscous and incompressible fluid, which steadily flows between two infinite vertical and parallel plane walls. At the entrance of the channel the fluid has an entrance velocity U_0 parallel to the vertical axis of the channel and the fluid temperature is T_0. The geometry of the problem, the boundary conditions, and the coordinate system are shown in Fig. 1. The variation of density with temperature is given by eq. (1) and the fluid rises in the duct driven by buoyancy forces and initial velocity U_0. Hence, the flow is due to difference in temperature and in the pressure gradient.

The flow being fully developed the following relations apply here:

$$v = 0,\ \partial v/\partial y = 0,\ \partial p/\partial y = 0,$$

where v is the velocity in the transversal direction and p is the pressure. Thus, from the continuity equation, we get

$$\partial u / \partial x = 0$$

so that

$$u = u(y).$$

Based on the fact that the flow is fully developed we can assume that $T = T(y)$. Under these assumptions the momentum and energy equations for the flow and heat transfer have the following form:

$$\nu \frac{d^2 u}{dy^2} - \frac{1}{\rho}\frac{dp}{dx} + g\beta\left(T - T_0\right) + g\gamma\left(T - T_0\right)^2 = 0 \tag{2}$$

$$\alpha \frac{d^2 T}{dy^2} + \frac{\nu}{c_p}\left(\frac{du}{dy}\right)^2 = 0 \tag{3}$$

subject to the boundary conditions

$$u(0) = 0, \quad u(L) = 0, \quad T(0) = T_1, \quad T(L) = T_2 \tag{4}$$

where p is the pressure, g is the gravitational acceleration, ν is the kinematic viscosity, α is the thermal diffusivity and c_p is the specific heat at constant pressure. The closure of the system (2) – (3) subject to the boundary conditions (4) is given by the mass flux conservation equation

$$U_0 = \frac{1}{L}\int_0^L u(y)dy \tag{5}$$

where L is the channel width.

In order to solve equations (2) and (3), we introduce the following non-dimensional variables

$$U = \frac{u}{U_0}, \quad X = \frac{xRe}{L}, \quad Y = \frac{y}{L}, \quad \theta = \frac{T - T_0}{T_2 - T_0}, \quad P = \frac{L^2}{\rho \nu^2} p \tag{6}$$

where $Re = U_0 L/\nu$ is the Reynolds number.

$$\frac{du}{dy} = \frac{d(U_0 U)}{dy} = U_0 \frac{dU}{dY}\frac{dY}{dy} = \frac{U_0}{L}\frac{dU}{dY};$$

$$\frac{d^2u}{dy^2} = \frac{d}{dy}\left(\frac{U_0}{L}\frac{dU}{dY}\right) = \frac{U_0}{L^2}\frac{d^2U}{dY^2}$$

$$\frac{dT}{dy} = \frac{d}{dy}\left(\theta(T_2 - T_0) + T_0\right) = (T_2 - T_0)\frac{d\theta}{dY}\frac{dY}{dy} = \frac{T_2 - T_0}{L}\frac{d^2\theta}{dY^2};$$

$$\frac{d^2T}{dy^2} = \frac{d}{dy}\left(\frac{T_2 - T_0}{L}\frac{d\theta}{dY}\right) = \frac{T_2 - T_0}{L^2}\frac{d\theta}{dY} \tag{6'}$$

$$\frac{dp}{dx} = \frac{d}{dx}\left(\frac{\rho\nu^2}{L^2}P\right) = \frac{\rho\nu^2}{L^2}\frac{dP}{dX}\frac{dX}{dx} = \frac{\mathrm{Re}\,\rho\nu^2}{L^3}\frac{dP}{dX} = \frac{\rho\nu U_0}{L^2}\frac{dP}{dX}$$

Using (6) in (2) and (3) we obtain:

$$\nu\frac{U_0}{L^2}\frac{d^2U}{dY^2} - \frac{1}{\rho}\frac{\rho\nu U_0}{L^2}\frac{dP}{dX} + g\beta(T_2 - T_0)\theta + g\gamma(T_2 - T_0)^2\theta^2 = 0$$

$$\alpha\frac{T_2 - T_0}{L^2}\frac{d^2\theta}{dY^2} + \frac{\nu}{c_p}\frac{U_0}{L^2}\left(\frac{dU}{dY}\right)^2 = 0$$

$$\frac{d^2U}{dY^2} - \frac{dP}{dX} + \frac{g\beta(T_2 - T_0)L^2}{U_0\nu}\theta + \frac{g\gamma(T_2 - T_0)^2 L^2}{U_0\nu}\theta^2 = 0$$

$$\frac{d^2\theta}{dY^2} + \frac{\nu U_0^{\,2}}{\alpha(T_2 - T_0)c_p}\left(\frac{dU}{dY}\right)^2 = 0$$

$$\frac{d^2U}{dY^2} - \frac{dP}{dX} + \lambda\theta + \lambda^*\theta^2 = 0 \tag{7}$$

$$\frac{d^2\theta}{dY^2} + Br\left(\frac{dU}{dY}\right)^2 = 0 \tag{8}$$

where dP/dX in equation (7) should be constant and where

$$Br = PrEc = \frac{\nu}{\alpha}\frac{U_0^{\,2}}{c_p(T_2 - T_0)} = \frac{\nu U_0^{\,2}}{\alpha c_p(T_2 - T_0)} \tag{9}$$

is the Brinkman number, see [9], Pr is the Prandtl number, Ec is the Eckert number and

$$\lambda = \frac{Gr}{Re} = \frac{g\beta(T_2 - T_0)L^2}{U_0\nu},\quad \lambda^* = \frac{Gr^*}{Re} = \frac{g\gamma(T_2 - T_0)^2 L^2}{U_0\nu},\quad Gr = \frac{g\beta(T_2 - T_0)L^3}{\nu^2},\quad Gr = \frac{g\beta(T_2 - T_0)^2 L^3}{\nu^2} \tag{10}$$

are the mixed convection parameter and the modified mixed convection parameter, respectively. It is worth mentioning that $\lambda^* > 0$, while $\lambda > 0$ if $T_0 < T_2$ (i.e. the fluid is cooler than the right wall) and $\lambda < 0$ if $T_0 > T_2$ (i.e. the fluid is hotter then the right wall because of the viscous dissipation), respectively.

Equations (7) and (8) are subject to the boundary conditions (4), which become in dimensionless form

$$U(0)=0,\quad U(1)=0,\quad \theta(0)=\frac{T_1-T_0}{T_2-T_0}=r_T,\quad \theta(1)=1 \tag{11}$$

and the conservation mass flux relation (5) takes the form

$$\int_0^1 U(Y)\,dY=1 \tag{12}$$

The physical quantities of interest in this problem are the skin friction coefficients C_f and the Nusselt numbers Nu, which are defined as

$$C_f=\frac{\mu}{\rho U_0^{\,2}}\left(\frac{du}{dy}\right)_{y=0,L},\quad Nu=-\frac{L}{(T_2-T_0)}\frac{dT}{dy}\bigg|_{y=0,L} \tag{13}$$

Using (6) and (13), we obtain

$$C_f\ Re=\left(\frac{dU}{dY}\right)_{Y=0,1},\quad Nu=-\left(\frac{d\theta}{dY}\right)_{Y=0,1} \tag{14}$$

It is worth mentioning that if the modified mixed parameter λ^* and the Brinkman number Br are zero (i.e. $\lambda^*=0, Br=0$) equations (7), (8) and (11) reduce to those obtained in [4] and it is possible to obtain an analytical solution of this problem.

References

1. Aung, W.: *Fully developed laminar free convection between vertical plates heated asymmetrically*. Int. J. Heat Mass Transfer, 15, 1972, p. 1577-1580.
2. Aung, W., Fletcher, L.S., Sernas, V.: *Developing laminar free convection between vertical flat plates with asymmetric heating.* Int. J. Heat Mass Transfer, 15, 1972, p. 2293-2308.
3. Aung, W., Worku, G.: *Developing flow and flow reversal in a vertical channel with asymmetric wall temperatures.* J. Heat Transfer, 108, 1986, p. 299-304.
4. Aung, W., Worku, G.: *Theory of fully developed, combined convection including flow reversal.* J. Heat Transfer, 108, 1986, p. 485-488.
5. Barletta, A.: *Analysis of combined forced and free flow in a vertical channel with viscous dissipation and isothermal-isoflux boundary conditions.* J. Heat Transfer, 121, 1999, p. 349-356.
6. Barletta, A.: *Fully developed mixed convection and flow reversal in a vertical rectangular duct with uniform wall heat flux.* Int. J. Heat Mass Transfer, 45, 2002, p. 641-654.
7. Boulama, K., Galanis, N.: *Analytical solution for fully developed mixed convection between parallel vertical plates with heat and mass transfer.* J. Heat Transfer, 126, 2004, p. 381-388.
8. Vajravelu, K., Sastri, K.S.: *Fully developed laminar free convection flow between two parallel vertical walls-I.* Int. J. Heat Mass Transfer, 20, pp. 655-660.
9. Barletta, A.: *Laminar mixed convection with viscous dissipation in a vertical channel.* Int. J. Heat Mass Transfer, 41, 1988, p. 3501-3513.