Elliptic and hyperbolic equations

1. Solve the heat equation problem in a rectangular square:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + 1 = 0, \quad (x, y) \in D = [0,1] \times [0,1]$$

$$T(x, y)\big|_{\partial D} = 0$$

2. Solve the wave equation:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0, \quad x \in [-5,5], \ t \geq 0$$

$$u(x,0) = \exp\!\left(1 - 5x^2\right)$$

Compare the result with the analytical solution $u(x,t) = \exp\!\left(1 - 5(x-t)^2\right)$.