Internal heat generation effects on the unsteady free convection in a square cavity filled with a porous medium

1. Introduction

Natural convective heat transfer in fluid-saturated porous media has occupied the centre stage in many fundamental heat transfer analyses and has received considerable attention over the last several decades. This interest has been due to its wide range of applications in, for example, packed sphere beds, high performance insulation for buildings, chemical catalytic reactors, grain storage and such geophysical problems as frost heave. Porous media are also of interest in relation to the underground spread of pollutants, solar power collectors, and to geothermal energy systems. Porous materials, such as sand and crushed rock, which when underground are saturated with water, which, under the influence of local pressure gradients, migrates and transports energy through the material. Literature concerning convective flow in porous media is abundant. Representative studies in this are may be found in the recent books by Nield and Bejan (2006), Ingham and Pop (2005), Vafai (2005), Bejan *et al.* (2004) and Pop and Ingham (2001).

Natural convection in an enclosure in which internal heat generation is present is of prime importance in certain technological applications. Examples are post-accident heat removal in nuclear reactors and geophysical problems associated with the underground storage of nuclear water, among others (Acharya and Goldstein, 1985; Ozoe and Maruo, 1987; Lee and Goldstein, 1988; Fusegi *et al*., 1992; Venkatachalappa and Subbaraya, 1993; Shim and Hyun, 1997; Hossain and Wilson, 2002; Hossain and Rees, 2005). Natural convection heat transfer in a cavity saturated with porous media in the presence of a magnetic field is a new branch of thermo-fluid mechanics. The heat transport phenomenon can be described by means of the hydrodynamics, the convective heat transfer mechanism and the electromagnetic field as they have a symbiotic relationship.

The present study investigates the effects heat generation on the unsteady free convection in a square enclosure filled with a porous medium saturated by a viscous fluid.

2. Mathematical model

Consider the unsteady natural convection flow in a rectangular cavity filled with an electrically, conducting fluid-saturated porous medium and internal heat generation. We assume that the enclosure is permeated by a uniform inclined magnetic field. The geometry and the Cartesian coordinate system, and the boundary conditions, are schematically shown in Fig. 1, where the dimensional coordinates x and y are measured along the horizontal bottom wall and normal to it along the left vertical wall, respectively.

The height of the cavity is denoted by L. It is assumed that the vertical walls are maintained at constant temperatures T_c and T_h, while the horizontal walls are adiabatic. We also take into account the effect of uniform heat generation in the flow region. The constant volumetric rate of heat generation is $q_0''' [W/m^3]$. It is also assumed that the effect of buoyancy is included through the well-known Boussinesq approximation. The resulting convective flow is governed by the combined mechanism of the driven buoyancy force and internal heat generation.

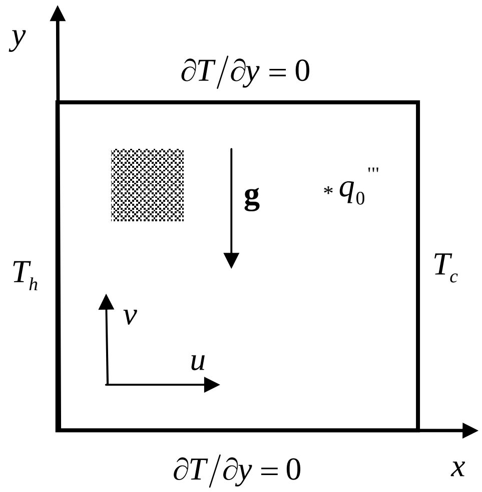

Figure 1.Geometry of the problem and the co-ordinate system, and the boundary conditions.

Under the above assumptions, the conservation equations for mass, Darcy and energy are given by

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\mathbf{V} = \frac{K}{\mu}\left(-\nabla p + \rho \mathbf{g}\right) \tag{2}$$

$$\frac{\partial T}{\partial t'} + (\mathbf{V} \cdot \nabla)T = \alpha_m \nabla^2 T + \frac{q_0'''}{\rho_0 c_p} \tag{3}$$

$$\rho = \rho_0\left[1 - \beta\left(T - T_0\right)\right] \tag{4}$$

where $\mathbf{V}$ is the velocity vector, T is the fluid temperature, p is the pressure, $\mathbf{g}$ is the gravitational acceleration, K is the permeability of the porous medium, α_m is the effective thermal diffusivity, ρ is the density, μ is the dynamic viscosity, β is the coefficient of thermal expansion, c_p is the specific heat at constant pressure, ρ_0 is the reference density

Eliminating the pressure term in Eq. (2) in the usual way, the governing equations (1) to (3) can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{gK\beta}{\nu}\frac{\partial T}{\partial x} \tag{8}$$

$$\frac{\partial T}{\partial t'} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{q_0'''}{\rho c_p} \tag{9}$$

and are subjected to the boundary conditions

$$
\begin{aligned}
&u = 0,\ T = T_h \quad \text{at} \quad x = 0, \quad 0 < y < L \\
&u = 0,\ T = T_c \quad \text{at} \quad x = L, \quad 0 < y < L \\
&v = 0,\ \frac{\partial T}{\partial y} = 0, \quad \text{at} \quad y = 0 \quad \text{and} \quad y = L, \quad 0 < x < L
\end{aligned}
\tag{10}
$$

where υ is the kinematic viscosity. Further, we introduce the following non-dimensional variables

$$
t = \frac{\alpha_m}{L^2} t', \quad X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{L}{\alpha_m} u, \quad V = \frac{L}{\alpha_m} v, \quad \theta = \frac{T - T_0}{T_h - T_c} \tag{11}
$$

where $T_0 = (T_h + T_c)/2$ is the characteristic temperature. Introducing the stream function ψ defined as $U = \partial\psi / \partial Y$ and $V = -\partial\psi / \partial X$, and using Eq. (11) in Eqs. (7) - (9), we obtain the following partial differential equations in non-dimensional form:

$$
\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -Ra \frac{\partial \theta}{\partial X} \tag{12}
$$

$$
\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{Ra_I}{Ra} \tag{13}
$$

subject to the boundary conditions

$$
\begin{aligned}
&\psi = 0, \quad \theta = 1/2, \quad \text{at} \quad X = 0, \quad 0 < Y < 1 \\
&\psi = 0, \quad \theta = -1/2, \quad \text{at} \quad X = 1, \quad 0 < Y < 1 \\
&\psi = 0, \quad \frac{\partial \theta}{\partial Y} = 0, \quad \text{at} \quad Y = 0 \quad \text{and} \quad Y = 1, \quad 0 < X < 1
\end{aligned}
\tag{14}
$$

where $Ra = gK\beta(T_h - T_c)L / \alpha_m \nu$ is the Rayleigh number and $Ra_I = g q_0''' K\beta L^3 / k\alpha_m \nu$ is the heat generation parameter.

Once we know the numerical values of the temperature function we may obtain the rate of heat flux from each of the vertical walls. The non-dimensional heat transfer rate, per unit length in the depth-wise direction for the left vertical wall is given by

$$
Nu_Y = -\left(\frac{\partial \theta}{\partial X}\right)_{X=0}, \quad Nu = -\int_0^1 \left(\frac{\partial \theta}{\partial X}\right)_{X=0} dY \tag{15}
$$

3 Results and discussion

We use a non-uniform grid and in the absence of internal heat generation effect we obtain:

Discretizare:

$$\begin{cases} \dfrac{\partial^2 \psi}{\partial x^2} + \dfrac{\partial^2 \psi}{\partial y^2} = -Ra\,\dfrac{\partial \theta}{\partial x} & \text{(ec. momentului)} \\[2ex] \dfrac{\partial \theta}{\partial t} + \dfrac{\partial \psi}{\partial y}\dfrac{\partial \theta}{\partial x} - \dfrac{\partial \psi}{\partial x}\cdot\dfrac{\partial \theta}{\partial y} = \dfrac{\partial^2 \theta}{\partial x^2} + \dfrac{\partial^2 \theta}{\partial y^2} + Q & \text{(ec. energiei)} \\[2ex] \psi|_{\partial D} = 0; \quad \theta = 1/2 \text{ la } X = 0; \quad \theta = -1/2 \text{ la } X = 1; \quad \dfrac{\partial \theta}{\partial y} = 0 \text{ la } Y = 0, 1. \end{cases}$$

Grid Variabil:

Δx_1 Δx_2 Δx_{i-1} Δx_i

$i=1$ $i=2$ $i=3$ ... $i-1$ i $i+1$

$$\left(\frac{\partial f}{\partial x}\right)_i \simeq \frac{f_{i+1} - f_{i-1}}{\Delta x_i + \Delta x_{i-1}}$$

$$\left(\frac{\partial^2 f}{\partial x^2}\right)_i \simeq \frac{2\Delta x_{i-1} f_{i+1} - 2 f_i(\Delta x_{i-1} + \Delta x_i) + 2\Delta x_i f_{i-1}}{\Delta x_i\, \Delta x_{i-1}\, (\Delta x_i + \Delta x_{i-1})}$$

Avem: Momentul:

$$\frac{2\Delta x_{i-1}\, \psi_{i+1,j} - 2\psi_{ij}(\Delta x_i + \Delta x_{i-1}) + 2\Delta x_i\, \psi_{i-1,j}}{\Delta x_i\, \Delta x_{i-1}\, (\Delta x_i + \Delta x_{i-1})} +$$

$$+ \frac{2\Delta y_{j-1}\psi_{i,j+1} - 2\psi_{ij}(\Delta y_j + \Delta y_{j-1}) + 2\Delta y_j\, \psi_{i,j-1}}{\Delta y_j\, \Delta y_{j-1}\, (\Delta y_j + \Delta y_{j-1})} = -Ra\, \frac{\theta_{i+1,j} - \theta_{i-1,j}}{\Delta x_i + \Delta x_{i-1}} \qquad (a)$$

Energia:

implicit în direcția X:

$$\frac{\theta_{ij}^{(m+1/2)} - \theta_{ij}^{(m)}}{\left(\frac{\Delta t}{2}\right)} + \frac{\psi_{i,j+1}^{(m)} - \psi_{i,j-1}^{(m)}}{\Delta y_j + \Delta y_{j-1}} \cdot \frac{\theta_{i+1,j}^{(m+1/2)} - \theta_{i-1,j}^{(m+1/2)}}{\Delta x_i + \Delta x_{i-1}} - \frac{\psi_{i+1,j}^{(m)} - \psi_{i-1,j}^{(m)}}{\Delta x_i + \Delta x_{i-1}} \cdot \frac{\theta_{i,j+1}^{(m)} - \theta_{i,j-1}^{(m)}}{\Delta y_j + \Delta y_{j-1}} =$$

$$= \frac{2\Delta x_{i-1}\, \theta_{i+1,j}^{(m+1/2)} - 2(\Delta x_i + \Delta x_{i-1})\, \theta_{ij}^{(m+1/2)} + 2\Delta x_i\, \theta_{i-1,j}^{(m+1/2)}}{\Delta x_i\, \Delta x_{i-1}\, (\Delta x_i + \Delta x_{i-1})} +$$

$$+ \frac{2\Delta y_{j-1}\, \theta_{i,j+1}^{(m)} - 2(\Delta y_{j-1} + \Delta y_j)\, \theta_{ij}^{(m)} + 2\Delta y_j\, \theta_{i,j-1}^{(m)}}{\Delta y_j\, \Delta y_{j-1}\, (\Delta y_j + \Delta y_{j-1})} + Q. \qquad (b)$$

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implicit în direcția Y:

$$\frac{\theta_{i,j}^{m+1} - \theta_{ij}^{(m+1/2)}}{\left(\frac{\Delta t}{2}\right)} + \frac{\psi_{i,j+1}^{(m)} - \psi_{i,j-1}^{(m)}}{\Delta y_j + \Delta y_{j-1}} \cdot \frac{\theta_{i+1,j}^{(m+1/2)} - \theta_{i-1,j}^{(m+1/2)}}{\Delta x_i + \Delta x_{i-1}} - \frac{\psi_{i+1,j}^{(m)} - \psi_{i-1,j}^{(m)}}{\Delta x_i + \Delta x_{i-1}} \cdot \frac{\theta_{i,j+1}^{(m+1)} - \theta_{i,j-1}^{(m+1)}}{\Delta y_j + \Delta y_{j-1}} =$$

$$= \frac{2\Delta x_{i-1}\theta_{i+1,j}^{(m+1/2)} - 2(\Delta x_i + \Delta x_{i-1})\theta_{i,j}^{(m+1/2)} + 2\Delta x_i \theta_{i-1,j}^{(m+1/2)}}{\Delta x_i \cdot \Delta x_{i-1}\,(\Delta x_i + \Delta x_{i-1})} +$$

$$+ \frac{2\Delta y_{j-1}\theta_{i,j-1}^{(m+1)} - 2(\Delta y_j + \Delta y_{j+1})\theta_{i,j}^{(m+1)} + 2\Delta y_j \theta_{i,j-1}^{(m+1)}}{\Delta y_j\, \Delta y_{j-1}\,(\Delta y_j + \Delta y_{j-1})} + Q. \quad (c)$$

Algoritm:
1. Se rezolvă b)
2. Se rezolvă c)
3. Se rezolvă a)
(Se verifică dacă $|\psi - \psi_{old}| < \varepsilon$)
4. Se trece la următorul pas de timp.

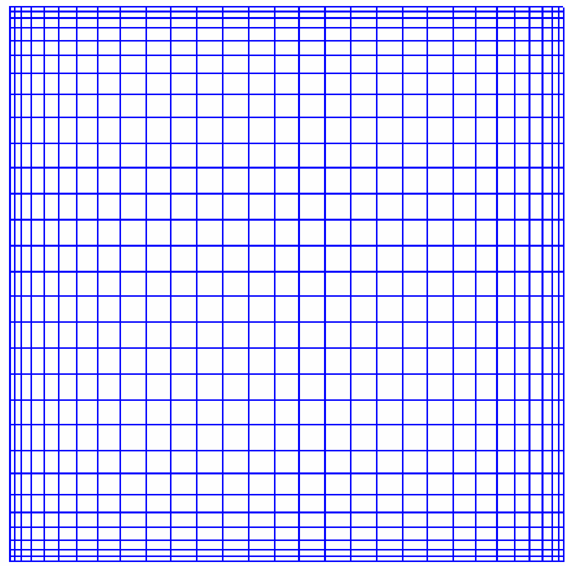

Fig. 2. Non-uniform grid.

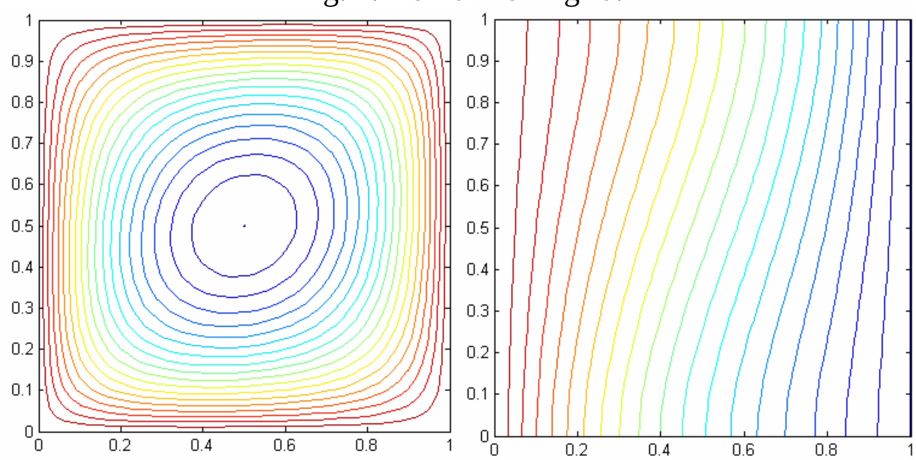

Fig. 3. Streamlines and isotherms for Ra = 10 and Q = 0 (steady state).

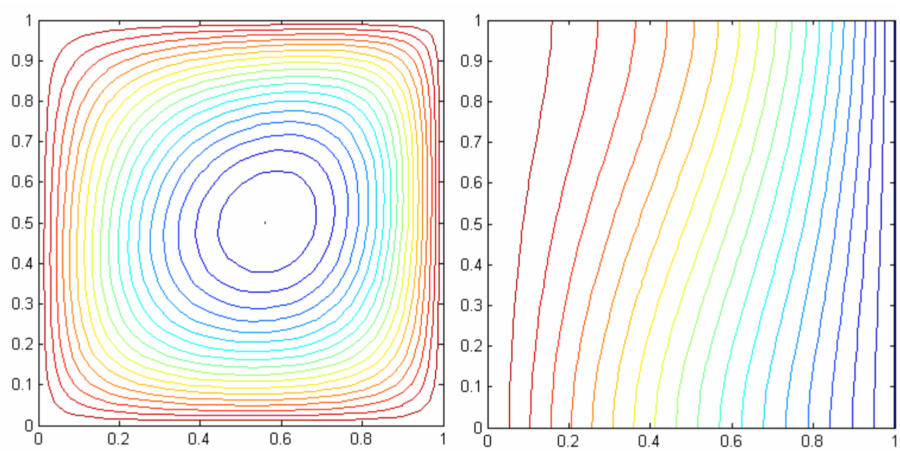

Fig. 4. Streamlines and isotherms for Ra = 10 and Q = 1 (steady state).

Table 1. Mean Nusselt number $\overline{Nu}$ for different grids at $Ra = 10^3$

Boundary layer thickness	Δx_1 - first step in b.l.	N_{bl} - number of nodes in b.l.	N – total number of nodes	$\overline{Nu}$ (Nusselt number)
0.1	0.00123	10	56	13.4523
0.1	0.00027	20	116	13.5982
0.2	0.00054	20	67	13.5521
0.2	0.00023	30	102	13.6027
0.3	0.00035	30	77	13.5822
0.3	0.00019	40	104	13.6085
0.4	0.00026	40	87	13.5968
0.4	0.00016	50	110	13.6131

Table 2. Comparison of the mean Nusselt number $\overline{Nu}$ for different values of Ra when the steady state is reached

Authors	Ra			
	10	100	1000	10000
Walker and Hosmy [20]		3.097	12.96	51
Bejan [21]		4.2	15.8	50.80
Beckerman et al. [22]		3.113		48.9
Gross et al. [23]		3.141	13.448	42.583
Manole and Lage [24]		3.118	13.637	48.117
Moya et al. [25]	1.065	2.801		
Batas and Pop [26]	1.079	3.16	14.06	48.33
Present results (110 x 110)	1.079	3.108	13.613	48.208

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