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# EXISTENCE AND CONVERGENCE RESULTS FOR PEROV CONTRACTION MAPPINGS

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**Abstract.** In this work we consider a contraction mapping of Perov type which maps a closed subset of a generalized complete metric space into the space. We show the existence of a unique fixed point which attracts all (inexact) iterates of the mapping uniformly on bounded sets.
**Key Words and Phrases**: Complete metric space, contraction mapping, fixed point, inexact iterate.
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## 1. Introduction

For more than fifty-five years now, there has been a lot of research activity regarding the fixed point theory of nonexpansive (that is, 1-Lipschitz) mappings. See, for example, [2, 4, 5, 8, 9, 10, 12, 13, 14, 15, 16, 20, 21, 23, 24, 25, 26, 27, 28, 33, 34, 35] and the references cited therein. This activity stems from Banach's classical theorem [1] concerning the existence of a unique fixed point for a strict contraction. It also concerns the convergence of (inexact) iterates of a nonexpansive mapping to one of its fixed points. Since that seminal result, many developments have taken place in this field including, in particular, studies of feasibility, common fixed point problems and variational inequalities, which find important applications in engineering, medical and the natural sciences [3, 6, 7, 11, 29, 30, 31, 34, 35].

In this work we consider a contraction mapping of Perov type which maps a closed subset of a generalized complete metric space into the space. The study of this class of mappings is an important topic in the fixed point theory [17, 18, 19, 22, 32]. We show the existence of a unique fixed point which attracts all (inexact) iterates of the mapping uniformly on bounded sets.

## 2. Main results

Let $R^n$ be an $n$-dimensional Euclidean space. In other words,

$$R^n = \{x = (x_1, \dots, x_n) : \ x_i \in R^1, \ i = 1, \dots, n\}.$$

Let

$$R^n_+ = \{x = (x_1, \dots, x_n) \in R^n : \ x_i \geq 0, \ i = 1, \dots, n\}$$

and $e = (1, 1, \dots, 1) \in R^n$. We say that $x = (x_1, \dots, x_n)$, $y = (y_1, \dots, y_n) \in R^n$ satisfy $x \le y$ if $x_i \le y_i$ for all $i = 1, \dots, n$. For each $x = (x_1, \dots, x_n) \in R^n$ set

$$\|x\|_1 = \sum_{i=1}^{n} |x_i|.$$

Let $Y$ be a nonempty set and let $S : Y \to Y$. We denote by $S^0$ the identity mapping in $Y$, set $S^1 = S$ and for every integer $i \ge 0$ define

$$S^{i+1} = S \circ S^i.$$

We suppose that the sum over an empty set is zero.

Assume that $X$ is a nonempty set and a function $d : X \times X \to R^n_+$ satisfies for each $x, y, z \in X$,

$$d(x, y) = 0 \text{ if and only if } x = y,$$

$$d(x, y) = d(y, x),$$

$$d(x, z) \le d(x, y) + d(y, z).$$

The pair $(X, d)$ is called a generalized metric space and $d$ is called a generalized metric [17, 18, 19, 22, 32].

For all $x, y \in X$ set

$$d(x, y) = (d_1(x, y), \dots, d_n(x, y)).$$

Fix $\theta \in X$. For each $x \in X$ and each $r \in R^n_+$ set

$$B(x, r) = \{y \in X : \ d(x, y) \le r\}.$$

We say that a sequence $\{x_i\}_{i=0}^{\infty} \subset X$ converges to $x_* \in X$ if

$$\lim_{i \to \infty} \|d(x_i, x_*)\|_1 = 0.$$

We say that $\{x_i\}_{i=0}^{\infty} \subset X$ is a Cauchy sequence if for each $\epsilon > 0$ there exists a natural number $n(\epsilon)$ such that for each pair of integers $p, m \ge n(\epsilon)$,

$$\|d(x_p, x_m)\|_1 \le \epsilon.$$

The generalized metric space $(X, d)$ is complete if every Cauchy sequence converges.

For all $x, y \in X$ set

$$\tilde{d}(x, y) = \|d(x, y)\|_1 = d_1(x, y) + \dots + d_n(x, y).$$

Clearly, $(X, \tilde{d})$ is a metric space and convergence in $(X, d)$ is equivalent to the convergence in $(X, \tilde{d})$.

We assume that the metric space $(X, \tilde{d})$ is complete.

Assume that $A : R^n_+ \to R^n_+$ and that the following properties hold:

(i) $A(0) = 0$ and $A$ is continuous at zero;

(ii) for each $z_1, z_2 \in R^n_+$ satisfying $0 \le z_1 \le z_2$,

$$A(z_1) \le A(z_2);$$

(iii) for each $z_1, z_2 \in R^n_+$,

$$A(z_1 + z_2) \le A(z_1 + z_2);$$

(iv) $A^k(e) \to 0$ as $k \to \infty$;

(v) $A(\lambda z) = \lambda A(z)$ for each $\lambda \geq 0$ and each $z \in R^n_+$.

Assume that $K$ is a nonempty closed subset of $X$ and $T : K \to X$ satisfies for each $x, y \in K$,

$$d(T(x), T(y)) \leq A(d(x, y)). \tag{2.1}$$

It is natural to call the mapping $T$ as a generalized contraction [24, 27]. This class of mappings was introduced in [17]. A particular case when $A$ is a linear mapping was introduced in [18].

It is easy to see that the following auxiliary result holds.

**Lemma 2.1.** *Let $x, y \in K$, $m \geq 1$ be an integer and let $T^m(x), T^m(y)$ exist. Then*

$$d(T^m(x), T^m(y)) \leq A^m(d(x, y)).$$

Set

$$\Delta_1 = \sup\{\|A^i(e)\|_1 : \ i = 1, 2, \dots\} \tag{2.2}$$

(see property (iv)).

In this paper we prove the following results.

**Theorem 2.2.** *Assume that for each $\epsilon > 0$ there exists $x_\epsilon \in K$ such that*

$$d(x_\epsilon, T(x_\epsilon)) \leq \epsilon e.$$

*Then the following assertions hold.*

*1. There exists a unique point $x_T \in K$ such that $T(x_T) = x_T$.*

*2. For each $\epsilon > 0$ there exists $\delta > 0$ such that if $x \in K$ satisfies $d(x, T(x)) \leq \delta e$, then $d(x, x_T) \leq \epsilon e$.*

**Theorem 2.3.** *Assume that $c > 0$ and that for each integer $m \geq 1$ there exists $\{x_i^{(m)}\}_{i=0}^m \subset K$ such that*

$$d(x_0^{(m)}, x_1^{(m)}) \leq ce \tag{2.2}$$

*and that for each integer $i \in \{0, \dots, m-1\}$,*

$$d(x_{i+1}^{(m)}, T(x_i^{(m)})) \leq m^{-1}e. \tag{2.3}$$

*Then there exists a unique point $x_T \in K$ such that $T(x_T) = x_T$.*

**Theorem 2.4.** *Assume that $x_T \in K$ satisfies $T(x_T) = x_T$ and $c, \epsilon > 0$. Then there exist $\delta \in (0, \epsilon)$ and a natural number $n_0$ such that for each integer $m \geq n_0$ and each sequence $\{x_i\}_{i=0}^m \subset K$ which satisfies*

$$d(x_0, x_T) \leq ce,$$

$$d(x_{i+1}, T(x_i)) \leq \delta, \ i = 0, \dots, m-1$$

*the inequality*

$$d(x_i, x_T) \leq \epsilon e$$

*holds for all integers $i = n_0, \dots, m$.*

**Theorem 2.5.** *Assume that $x_T \in K$ satisfies $T(x_T) = x_T$ and $\epsilon > 0$. Then there exists $\delta \in (0, \epsilon)$ such that for each integer $k \geq 1$ and each sequence $\{x_i\}_{i=0}^k \subset K$ which satisfies*

$$d(x_0, x_T) \leq \delta e,$$
$$d(x_{i+1}, T(x_i)) \leq \delta,\ i = 0, \dots, k-1$$

*the inequality*

$$d(x_i, x_T) \leq \epsilon e$$

*holds for all integers $i = 0, \dots, k$.*

## 3. Proof of Theorem 2.2

By property (iv), there exists a natural number $k_* > 4$ such that for each integer $k \geq k_*$,

$$A^k(e) \leq (8n)^{-1} e. \tag{3.1}$$

**Proposition 3.1.** *Let $\epsilon \in (0, 1)$, a positive number $\delta$ satisfy*

$$\delta(2k_* + 1) \sum_{i=0}^{k_*-1} A^i(e) \leq (4n)^{-1} \epsilon e \tag{3.2}$$

*and let $\{x_i\}_{i=0}^{k_*}$, $\{y_i\}_{i=0}^{k_*} \subset K$ satisfy*

$$d(x_0, T(x_0)) \leq \delta e,\ d(y_0, T(y_0)) \leq \delta e \tag{3.3}$$

*and for each $i = 0, \dots, k_* - 1$*

$$d(x_{i+1}, T(x_i)) \leq \delta e,\ d(y_{i+1}, T(y_i)) \leq \delta e. \tag{3.4}$$

*Then*

$$d(x_0, y_0) \leq \epsilon e.$$

*Proof.* Assume that the proposition is not true. Then

$$\tilde{d}(x_0, y_0) > \epsilon. \tag{3.5}$$

In view of (3.3) and (3.4),

$$\begin{aligned} d(x_0, x_1) &\leq d(x_0, T(x_0)) + d(T(x_0), x_1) \leq 2\delta e, \\ d(y_0, y_1) &\leq d(y_0, T(y_0)) + d(T(y_0), y_1) \leq 2\delta e. \end{aligned} \tag{3.6}$$

By (2.1), (3.4) and (3.6),

$$\begin{aligned} d(x_1, x_2) &\leq d(x_1, T(x_0)) + d(T(x_0), T(x_1)) + d(T(x_1), x_2) \\ &\leq 2\delta e + A(d(x_0, x_1)) \leq 2\delta e + A(2\delta e), \\ d(y_1, y_2) &\leq d(y_1, T(y_0)) + d(T(y_0), T(y_1)) + d(T(y_1), y_2) \\ &\leq 2\delta e + A(d(y_0, y_1)) \leq 2\delta e + A(2\delta e). \end{aligned} \tag{3.7}$$

We show that for all $p = 0, \dots, k_* - 1$,

$$d(x_p, x_{p+1}) \leq \sum_{i=0}^{p} A^i(2\delta e), \tag{3.8}$$

$$d(y_p, y_{p+1}) \le \sum_{i=0}^{p} A^i(2\delta e). \tag{3.9}$$

In view of (3.6) and (3.7), equations (3.8) and (3.9) hold for $p = 0, 1$.

Assume that $p \in \{1, \ldots, k_* - 2\}$ and that (3.8) and (3.9) hold. It follows from (2.1), (3.4) and (3.8) that

$$\begin{aligned} d(x_{p+1}, x_{p+2}) &\le d(x_{p+1}, T(x_p)) + d(T(x_p), T(x_{p+1})) + d(T(x_{p+1}), x_{p+2}) \\ &\le 2\delta e + A(d(x_p, x_{p+1})) \\ &\le 2\delta e + A(\sum_{i=0}^{p} A^i(2\delta e)) \le \sum_{i=0}^{p+1} A^i(2\delta e). \end{aligned}$$

Analogously, we show that

$$d(y_{p+1}, y_{p+2}) \le \sum_{i=0}^{p+1} A^i(2\delta e).$$

Thus the assumption made for $p$ also holds for $p+1$. Thus we showed that (3.8) and (3.9) hold for all $p = 0, \ldots, k_* - 1$. Equations (3.8) and (3.9) imply that

$$d(x_0, x_{k_*}) \le \sum_{p=0}^{k_*-1} d(x_p, x_{p+1}) \le \sum_{p=0}^{k_*-1} (\sum_{i=0}^{p} A^i(2\delta e)) \le k_* \sum_{i=0}^{k_*-1} A^i(2\delta e) \tag{3.10}$$

and analogously

$$d(y_0, y_{k_*}) \le k_* \sum_{i=0}^{k_*-1} A^i(2\delta e) \tag{3.11}$$

By (2.1) and (3.4),

$$\begin{aligned} d(x_1, y_1) &\le d(x_1, T(x_0)) + d(T(x_0), T(y_0)) + d(T(y_0), y_1) \\ &\le 2\delta e + A(d(x_0, y_0)). \end{aligned} \tag{3.12}$$

We show that for all $p = 1, \ldots, k_*$,

$$d(x_p, y_p) \le A^p(d(x_0, y_0)) + \sum_{i=0}^{p-1} A^i(2\delta e). \tag{3.13}$$

In view of (3.12) equation (3.13) holds for $p = 1$.

Assume that $p \in \{1, \ldots, k_* - 1\}$ and (3.13) holds. It follows from (2.1), (3.4) and (3.13) that

$$\begin{aligned} d(x_{p+1}, y_{p+1}) &\le d(x_{p+1}, T(x_p)) + d(T(x_p), T(y_p)) + d(T(y_p), y_{p+1}) \\ &\le 2\delta e + A(d(x_p, y_p)) \le A^{p+1}(d(x_0, y_0)) + \sum_{i=0}^{p} A^i(2\delta e) \end{aligned}$$

and (3.13) is true for $p+1$ too. Thus (3.13) holds for all integers $p = 0, \ldots, k_*$ and

$$d(x_{k_*}, y_{k_*}) \le A^{k_*}(d(x_0, y_0)) + \sum_{i=0}^{k_*-1} A^i(2\delta e). \tag{3.14}$$

By (3.1), (3.2), (3.10), (3.11) and (3.14),

$$\begin{aligned} d(x_0, y_0) &\leq d(x_0, x_{k_*}) + d(x_{k_*}, y_{k_*}) + d(y_{k_*}, y_0) \\ &\leq 2k_* \sum_{i=0}^{k_*-1} A^i(2\delta e) + \sum_{i=0}^{k_*-1} A^i(2\delta e) + A^{k_*}(d(x_0, y_0)) \\ &\leq 2\delta(2k_* + 1) \sum_{i=0}^{k_*-1} A^i(e) + \tilde{d}(x_0, y_0)A^{k_*}(e) \\ &\leq (2n)^{-1}\epsilon e + (4n)^{-1}\tilde{d}(x_0, y_0)e, \tilde{d}(x_0, y_0) \\ &\leq 2^{-1}\epsilon + 4^{-1}\tilde{d}(x_0, y_0) \end{aligned}$$

and $\tilde{d}(x_0, y_0) \leq \epsilon$. Proposition 3.1 is proved.

*Proof of Theorem 2.2.* For each integer $k \geq 1$ there exists $x_k \in K$ such that

$$d(x_k, T(x_k)) \leq k^{-1}e. \tag{3.15}$$

By Proposition 3.1, $\{x_k\}_{k=1}^{\infty}$ is a Cauchy sequence in $(X, \tilde{d})$. There exists

$$x_T = \lim_{k\to\infty} x_k. \tag{3.16}$$

By (3.15) and (3.16), for each integer $k \geq 1$,

$$\begin{aligned} d(x_T, T(x_T)) &\leq d(x_T, x_k) + d(x_k, T(x_k)) + d(T(x_k), T(x_T)) \\ &\leq d(x_T, x_k) + d(x_k, T(x_k)) + A(d(x_k, T(x_k)) \to 0 \end{aligned}$$

as $k \to \infty$. Thus $x_T = T(x_T)$. Now Assertion 2 follows from Proposition 3.1. The uniqueness of the fixed point of $T$ follows from Assertion 2. Theorem 2.2 is proved.

## 4. Proof of Theorem 2.3

Let $\epsilon \in (0, 1)$. In view of Theorem 2.2, it is sufficient to show that there exists $x_\epsilon \in K$ such that

$$d(x_k, T(x_k)) \leq \epsilon e.$$

Property (iv) implies that there exists a natural number $n_0 > 4$ such that

$$cA^{n_0-1}(e) < 8^{-1}\epsilon e. \tag{4.1}$$

Choose an integer

$$m > n_0 + 2$$

such that

$$m^{-1} \sum_{i=0}^{n_0-1} A^i(e) < 8^{-1}\epsilon e. \tag{4.2}$$

We show that for all integers $p = 1, \dots, m - 1$,

$$d(x_p^{(m)}, T(x_p^{(m)})) \leq cA^p(e) + 2 \sum_{i=0}^{p-1} A^i(m^{-1}e) - m^{-1}e. \tag{4.3}$$

By (2.1)-(2.3),

$$\begin{aligned} d(x_1^{(m)}, T(x_1^{(m)})) &\le d(x_1^{(m)}, T(x_0^{(m)})) + d(T(x_0^{(m)}), T(x_1^{(m)})) \\ &\le m^{-1}e + A(d(x_0^{(m)}, x_1^{(m)})) \le m^{-1}e + A(ce) \end{aligned}$$

and (4.3) holds for $p = 1$.
Assume that $p \in \{1, \dots, m-2\}$ and that (4.3) holds. By (2.1), (2.3) and (4.3),

$$\begin{aligned} d(x_{p+1}^{(m)}, T(x_{p+1}^{(m)})) &\le d(x_{p+1}^{(m)}, T(x_p^{(m)})) + d(T(x_p^{(m)}), T(x_{p+1}^{(m)})) \\ &\le m^{-1}e + A(d(x_p^{(m)}, x_{p+1}^{(m)})) \\ &\le m^{-1}e + A(d(x_p^{(m)}, T(x_p^{(m)})) + d(T(x_p^{(m)}), x_{p+1}^{(m)})) \\ &\le m^{-1}e + A(d(T(x_p^{(m)}), x_p^{(m)}) + m^{-1}e) \\ &\le m^{-1}e + A(2\sum_{i=0}^{p-1} A^i(m^{-1}e)) + A^{p+1}(ce) \\ &= cA^{p+1}(e) + 2\sum_{i=0}^{p} A^i(m^{-1}e) - m^{-1}e \end{aligned}$$

and (4.3) holds for $p + 1$ too. Thus (4.3) holds for all $p = 1, \dots, m-1$. In particular for $p = n_0 - 1$, in view of (4.1) and (4.2),

$$d(x_{n_0-1}^{(m)}, T(x_{n_0-1}^{(m)})) \le cA^{n_0-1}(e) + 2\sum_{i=0}^{n_0-1} A^i(m^{-1}e) < 8^{-1}\epsilon e + 8^{-1}\epsilon e < \epsilon e.$$

This completes the proof of Theorem 2.3.

## 5. Auxiliary results for Theorems 2.4 and 2.5

**Lemma 5.1.** *Assume that $x_T \in K$ satisfies*

$$T(x_T) = x_T, \tag{5.1}$$

*$\delta \in (0, 1)$, $m \ge 1$ is an integer and $\{x_i\}_{i=0}^m \subset K$ satisfies*

$$d(x_{i+1}, T(x_i)) \le \delta,\ i = 0, \dots, m-1. \tag{5.2}$$

*Then for all $p = 0, \dots, m$,*

$$d(x_p, x_T) \le A^p(d(x_0, x_T)) + \sum_{i=0}^{p-1} A^i(\delta e). \tag{5.3}$$

*Proof.* Clearly, (5.3) holds for $p = 0$. By (2.1), (5.1) and (5.2),

$$d(x_1, x_T) \le d(x_T, T(x_0)) + d(T(x_0), x_1) \le A(d(x_T, x_0)) + \delta e$$

and (5.3) holds for $p = 1$.

Assume that $p \in \{0, \dots, m-1\}$ and (5.3) holds. It follows from (2.1), (5.2) and (5.3) that

$$\begin{aligned} d(x_{p+1}, x_T) &\le d(x_{p+1}, T(x_p)) + d(T(x_p), x_T) \\ &\le \delta e + A(A^p(d(x_0, x_T)) + \sum_{i=0}^{p-1} A^i(\delta e)) \\ &\le A^{p+1}(d(x_0, x_T)) + \sum_{i=0}^{p} A^i(\delta e) \end{aligned}$$

and (5.3) holds for $p+1$. This completes the proof of Lemma 5.1.

**Lemma 5.2.** *Assume that* $x_T \in K$ *satisfies*

$$T(x_T) = x_T,$$

$c > 0$ *and* $\epsilon \in (0, 1)$. *Then there exist a natural number* $k$ *and* $\delta \in (0, \epsilon)$ *such that for each finite sequence* $\{x_i\}_{i=0}^k \subset K$ *which satisfies*

$$d(x_0, x_T) \le ce, \tag{5.4}$$

$$d(x_{i+1}, T(x_i)) \le \delta e, \ i = 0, \dots, k-1 \tag{5.5}$$

*the inequality*

$$d(x_k, x_T) \le \epsilon e.$$

*holds.*

*Proof.* Property (iv) implies that there exists a natural number $k$ such that

$$cA^k(e) < 4^{-1}\epsilon e. \tag{5.6}$$

Choose a positive number $\delta \in (0, \epsilon)$ such that

$$\delta \sum_{i=0}^{k-1} A^i(e) \le 4^{-1}\epsilon e. \tag{5.7}$$

Assume that $\{x_i\}_{i=0}^k \subset K$ satisfies (5.4) and (5.5). By Lemma 5.1 and (5.4)-(5.7),

$$\begin{aligned} d(x_k, x_T) &\le A^k(d(x_0, x_T)) + \sum_{i=0}^{k-1} A^i(\delta e) \\ &\le A^k(ce) + \delta \sum_{i=0}^{k-1} A^i(e) \le \epsilon e. \end{aligned}$$

Lemma 5.2 is proved.

## 6. Proofs of Theorem 2.4 and 2.5

*Proof of Theorem 2.5.* We may assume that $\epsilon \in (0,1)$. Properties (iv) and (v) imply that there exists

$$\epsilon_1 \in (0,\epsilon)$$

such that

$$A^i(\epsilon_1 e) \le 8^{-1}\epsilon e,\ i = 0,1,\dots. \tag{6.1}$$

By property (iv), there exists an integer $k_0 > 1$ such that

$$A^{k_0}(e) \le 8^{-1}\epsilon_1 e. \tag{6.2}$$

Choose $\delta \in (0,\epsilon_1)$ such that

$$\delta \sum_{i=0}^{k_0} A^i(e) \le 8^{-1}\epsilon_1 e. \tag{6.3}$$

Assume that $m \ge 1$ is an integer, $\{x_i\}_{i=0}^m \subset K$,

$$d(x_0, x_T) \le \delta e, \tag{6.4}$$

and that for all $i = 0,\dots,m-1$,

$$d(x_{i+1}, T(x_i)) \le \delta. \tag{6.5}$$

Assume that $j \in \{0,\dots,m\}$,

$$j + k_0 \le m \tag{6.6}$$

and that

$$d(x_{j_0}, x_T) \le e. \tag{6.7}$$

Set

$$y_i = x_{i+j_0},\ i = 0,\dots,k_0. \tag{6.8}$$

Lemma 5.1 and equations (6.2), (6.3), (6.7) and (6.8) imply that

$$d(x_{j+k_0}, x_T) = d(y_{k_0}, x_T) \le A^{k_0}(d(y_{k_0}, x_T)) + \sum_{i=0}^{k_0-1} A^i(\delta e) \le 8^{-1}\epsilon_1 e + 8^{-1}\epsilon_1 e.$$

Thus we have shown that the following property holds:

(a) if $j \in \{0,\dots,m\}$ satisfies (6.6) and (6.7), then $d(x_{j+k_0}, x_T) \le 4^{-1}\epsilon_1 e$.

Property (a) and (6.4) imply that the following property holds:

(b) if an integer $s \ge 0$ satisfies $sk_0 \le m$, then $d(x_{sk_0}, x_T) \le 4^{-1}\epsilon_1 e$.

Assume that $p \in \{1,\dots,m\}$. There exists an integer $s \ge 0$ such that

$$sk_0 \le p < (s+1)k_0. \tag{6.9}$$

Property (b) implies that

$$d(x_{sk_0}, x_T) \le 4^{-1}\epsilon_1 e. \tag{6.10}$$

Lemma 5.1, (6.1), (6.3), (6.5), (6.9) and (6.10) imply that

$$d(x_p, x_T) \leq A^{p-sk_0}(d(x_{sk_0}, x_T)) + \sum_{i=0}^{k_0} A^i(\delta e)$$
$$\leq 4^{-1}\epsilon_1 A^{p-s_0}(e) + (\epsilon_1/8)e \leq \epsilon.$$

Theorem 2.5 is proved.

Theorem 2.4 follows from Theorem 2.5 and Lemma 3.2.

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