

STRONG CONVERGENCE THEOREM FOR A GENERAL SYSTEM OF VARIATIONAL INEQUALITIES, EQUILIBRIUM PROBLEMS, AND FIXED POINT PROBLEMS

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Abstract. A new iterative scheme is proposed for finding a common element of the solution set of a general system of variational inequalities, the solution set of an equilibrium problem and the common fixed point set of a countable family of nonexpansive mappings in a real Hilbert space. Under some suitable conditions imposed on the parameters, a strong convergence theorem is proved. Moreover, a numerical result is given to show the effectiveness of the scheme.

Key Words and Phrases: Equilibrium problem, iterative method, fixed point.

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