

## SOLVABILITY AND OPTIMAL CONTROL OF SEMILINEAR FRACTIONAL EVOLUTION EQUATIONS WITH RIEMANN-LIOUVILLE FRACTIONAL DERIVATIVES

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**Abstract.** This paper is devoted to a class of semilinear Riemann-Liouville fractional evolution equations in Banach spaces. Using the Banach fixed point theorem and semigroup theory, we first establish an existence and uniqueness theorem of the mild solution, which improves the existing results in literature. Then, we consider an optimal control problem governed by semilinear fractional diffusion equations. The existence of optimal pairs and the compactness of the states are obtained. Moreover, the necessary optimality conditions of first order are derived.

**Key Words and Phrases:** Fractional evolution equation, optimal control, Riemann-Liouville derivatives, mild solution, optimal pair, necessary optimality conditions.

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