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COMMON FIXED POINT THEOREMS FOR MULTIVALUED MAPPINGS VIA CONE-VALUED MEASURE OF NONCOMPACTNESS

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Abstract. We obtain common fixed point theorems for a pair of condensing multivalued mappings with respect to a cone-valued measure of noncompactness under a semi-weakly isotone condition, and we apply it to the system of multivalued differential equations with deviating argument of the form

$$x'(t) \in f[t, x(t), x(h(t))] \text{ and} \tag{0.1}$$
$$x'(t) \in g[t, x(t), x(h(t))], x(0) = x_0, t \in [0, b]. \tag{0.2}$$

Key Words and Phrases: Multivalued equations, common fixed point, measure of noncompactness, semi-weakly isotone, condensing mutivalued mapping.
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