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STRONG CONVERGENCE OF INERTIAL EXTRAGRADIENT ALGORITHMS FOR SOLVING VARIATIONAL INEQUALITIES AND FIXED POINT PROBLEMS

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Abstract. This paper investigates two inertial extragradient algorithms for seeking a common solution to a variational inequality problem involving a monotone and Lipschitz continuous mapping and a fixed point problem with a demicontractive mapping in real Hilbert spaces. Our algorithms need to calculate the projection on the feasible set only once in each iteration. Moreover, they can work well without the prior information of the Lipschitz constant of the operator and do not contain any linesearch process. Strong convergence theorems of the suggested algorithms are established under suitable conditions. Some experiments are presented to illustrate the numerical efficiency of the suggested algorithms and compare them with some existing ones.
Key Words and Phrases: Variational inequality problem, fixed point problem, subgradient extragradient method, Tseng's extragradient method, inertial method, demicontractive mapping.
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