FIXED POINT AND NONLINEAR ERGODIC THEOREMS FOR SOME NEW TYPE SEMIGROUPS OF MAPPINGS

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Abstract. In the present paper, some new type of semigroups of mappings are introduced. Then, by using the theory of invariant means, fixed point theorem, and existence of nonspreading retraction for these semigroup are deduced and the illustrative examples are given. Also, weak convergence theorem of Mann's type, generalized Baillon's nonlinear ergodic theorem and strong convergence theorem of Halpern's type for such semigroups in the Hilbert space are considered. At the end, some applications are investigated.

Key Words and Phrases: Fixed point theorem, generalized nonlinear ergodic theorem, nonspreading retraction, convergence theorem.

2020 Mathematics Subject Classification: 47H05, 47H09, 47H20.

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