ITERATIVE CONSTRUCTION OF THE FIXED POINT OF SUZUKI'S GENERALIZED NONEXPANSIVE MAPPINGS IN BANACH SPACES

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Abstract. We approximate the fixed point of the Suzuki's generalized nonexpansive mappings via the Picard-Ishikawa hyprid iterative process, recently introduced by Okeke [18]. We prove some weak and strong convergence theorems of this type of mappings in the setting of uniformly convex Banach spaces. We apply our results in finding the solution of a mixed type Volterra-Fredholm functional nonlinear integral equation in Banach spaces. Finally, we give several numerical examples to validate our analytical results. Our results extend and improve several known results in literature, including the results of Okeke [18], Ullah and Arshad [30] and Craciun and Serban [7] among others.

Key Words and Phrases: Picard-Ishikawa hybrid iterative process, Suzuki's generalized nonexpansive mapping, weak convergence, Strong convergence, Volterra-Fredholm integral equation, data dependence, Banach space.

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