

**THE EXISTENCE AND COMPACTNESS OF THE SET  
OF SOLUTIONS FOR A 2-ORDER NONLINEAR  
INTEGRODIFFERENTIAL EQUATION IN  $N$  VARIABLES  
IN A BANACH SPACE**

LE THI PHUONG NGOC\* AND NGUYEN THANH LONG\*\*,\*\*

\*University of Khanh Hoa, 01 Nguyen Chanh Str., Nha Trang City, Vietnam  
E-mail: ngoc1966@gmail.com

\*\*Department of Mathematics and Computer Science, University of Science,  
227 Nguyen Van Cu Str., Dist. 5, Ho Chi Minh City, Vietnam

\*\*\*Vietnam National University Ho Chi Minh City, Vietnam  
E-mail: longnt2@gmail.com

**Abstract.** In this paper, by applying the fixed point theorem of Krasnosel'skii, we prove the existence and compactness of the set of solutions for a 2-order nonlinear integrodifferential equation in  $N$  variables in an arbitrary Banach space  $E$ . Here, an appropriate Banach space  $X_1$  for the above equation is defined and a sufficient condition for relatively compact subsets in  $X_1$  is proved. An example is given to verify the efficiency of the used method.

**Key Words and Phrases:** Nonlinear integrodifferential equation in  $N$  variables, the fixed point theorem of Krasnosel'skii.

**2020 Mathematics Subject Classification:** 45G10, 47H10, 47N20, 65J15.

**Acknowledgements.** The authors wish to express their sincere thanks to the editor and the referees for the valuable comments and remarks for the improvement of the paper. This research is funded by Vietnam National University HoChiMinh City (VNU-HCM) under grant number **B2020-18-01**.

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*Received: March 12, 2020; Accepted: August 27, 2021.*

