

## LARGE TIME BEHAVIOR OF SOLUTIONS TO A SYSTEM OF COUPLED NONLINEAR OSCILLATORS VIA A GENERALIZED FORM OF SCHAUDER-TYCHONOFF FIXED POINT THEOREM

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*Dedicated to Academician Dorin Ieșan on the occasion of his eighties anniversary*

**Abstract.** In this paper we investigate the stability of the null solution of a system of ODEs describing the motion of two coupled damped nonlinear oscillators. We also show that for any solution  $(x, y)$  of the system we have  $\lim_{t \rightarrow +\infty} x(t) = \lim_{t \rightarrow +\infty} \dot{x}(t) = \lim_{t \rightarrow +\infty} y(t) = \lim_{t \rightarrow +\infty} \dot{y}(t) = 0$ , for small initial data in the case when the uniqueness of solutions is not guaranteed. Our proofs are mainly based on a generalized form of Schauder-Tychonoff fixed point theorem. The theoretical results are illustrated with numerical simulations.

**Key Words and Phrases:** Coupled oscillators, uniform stability, asymptotic stability, fixed point theorem.

**2020 Mathematics Subject Classification:** 34D20, 47H10.

**Acknowledgements.** We are indebted to the reviewers for their valuable comments and suggestions which contributed to the improvement of the paper.

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*Received: June 11, 2021; Accepted: March 17, 2022.*

